University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MAT C34F 2013/14

Midterm Exam

Friday, November 1, 2013; 120 minutes

No books or calculators may be used

You may use any theorems stated in class, as long as you state them clearly and correctly.

1. **(25 points)** Is the function $f(z)$ defined by

$$f(z) = z^2\bar{z}$$

differentiable at $z = 0$? If you think so, give a proof and compute $\frac{df}{dz}$ at this value; if you think not, show why the complex derivative at 0 does not exist.

2. **(25 points)** Let γ denote the contour around the boundary of the unit disc $|z| \leq 1$, oriented counterclockwise. Evaluate the following integrals:

(a) $\int_\gamma \frac{1}{(z-3)(z-4)}dz$

(b) $\int_\gamma z|z|^4 dz$

(c) $\int_\gamma \frac{1}{z^2}dz$

3. **(25 points)** Let f be the function

$$f(z) = \frac{1}{4+z^2}$$

on the unit disk $\{z \in \mathbf{C} : |\mathbf{z}| \leq \mathbf{1}\}$.

(a) What is the maximum value of $|f(z)|$ on the disk?

(b) At what value(s) of z is the maximum value of $|f(z)|$ attained?

State all theorems you use.

4. **(25 points)**

(a) Compute the Laurent series at $z = 1$ for the following function:

$$f(z) = \frac{1}{z^2 - 1}$$

(b) Classify the singularities of the following functions, and state the orders of all zeroes and poles:

i. $\frac{1}{(z^2+1)\sin z}$

ii. $\frac{1}{z^3(z^2+1)}$