University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MAT C34F 2013/14

Problem Set #5

Due date: Thursday, November 28, 2013 at the beginning of class

Do the following problems.

1. By considering the integral
$$\int_{\gamma(0;1)} \frac{z}{(2z^4+5z^2+2)}dz,$$
prove that
$$\int_0^{2\pi} \frac{1}{1+8\cos^2\theta)}d\theta = 2\pi/3.$$

2. Evaluate
$$\lim_{R\to\infty}\int_{\Gamma_R} \frac{e^{iz}}{(z^4+z^3+z^2+z+1)^2}dz$$
$\Gamma_R(\theta) = Re^{i\theta}, 0 \le \theta \le \pi$ is a semicircle of radius R in the upper half plane (the notation is as in class).

3. A function f is holomorphic in C except for double poles at 1 and -1 of residues a and b respectively. It is also given that, for some constant K, $|z^2 f(z)| \le K$ for large $|z|$. Prove that $a+b=0$. Find f if $a=1$ and $f(2i)=f(-2i)=0$.

4. Prove $\int_{-\infty}^{\infty} \frac{1}{(x^2+x+1)^2}dx = \frac{4\pi}{3\sqrt{3}}$.

5. Prove $\int_0^\infty \frac{\sin^2 x}{x^2}dx = \pi/2$.

6. Prove that $\int_0^\infty \frac{(\log x)^2}{1+x^2}dx = \pi^3/8$.

7. Evaluate by contour integration
$$\int_0^\infty \frac{x^2dx}{(1+x^2)^2}$$