University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MAT C34F 2012/13

Problem Set #2

Due date: Thursday, September 26, 2013 at the beginning of class

1. For each arc C and function f find the value of

$$\int_C f(z)dz :$$

$f(z) = (z+2)/z$ and C is
(i) the semicircle $z = 2e^{i\theta}$ $(0 \le \theta \le \pi)$;
(ii) the circle $z = 2e^{i\theta}$ $(0 \le \theta \le 2\pi)$.

2. Show that if C is the boundary of the square with vertices at the points $z = 0$, $z = 1$, $z = 1+i$, $z = i$ and the orientation of C is counterclockwise, then

$$\int_C (3z+1)dz = 0.$$

3. Describe the image of the curve γ in the following cases.
(i) $\gamma(t) = 1 + ie^{it}$, $t \in [0, \pi]$
(ii) γ is the join of $[-1, 1]$, $[1, 1+i]$ and $[1+i, -1-i]$
(iii) γ is given by $\gamma(t) = e^{it}$ $(t \in [0, \pi])$ and $\gamma(t) = e^{-it}$ $(t \in [\pi, 2\pi])$.

4. Compute the integrals

(a) $\int_\gamma |z|^4 dz$,
(b) $\int_\gamma \mathrm{Re(z)}^2 dz$
(c) $\int_\gamma z^{-2}(z^4 - 1)dz$
(d) $\int_\gamma \sin(z)ds$
(e) $\int_\gamma z^{-1}(\bar{z} - 1/2)dz$

where $\gamma = \gamma(0; 1)$.

5. A function is holomorphic and real-valued in a region G. Prove f is constant. Is this true if G is an arbitrary open set?

6. Evaluate $\int_\gamma (1+z^2)^{-1}$ when γ is

   (i) $\gamma(0;2)$

   (ii) $\gamma(3i;\pi)$

   (You shouldn't have to perform big calculations to get the answer.)

7. Let γ be a polygonal path with initial point 0 and final point 1. What are all possible values of $\int_\gamma (1+z^2)^{-1} dz$?