2 Chapter 2: Topology

Definition 2.1 *A set is* connected *if it cannot be expressed as the union of two disjoint nonempty open sets.*

Definition 2.2 *A* region *is a nonempty open subset of* $\mathbf{C}$ *that is connected.*

Definition 2.3 *A* polygonally connected path *is the join of a collection of line segments* $[a_1, a_2], [a_2, a_3], \ldots, [a_{n-1}, a_n]$ *in the complex plane.*

Definition 2.4 *A subset* S *of* $\mathbf{C}$ *is* polygonally connected *if for any* $a, b \in S$ *there is a polygonally connected path in* S *with endpoints* a *and* b.

Theorem 2.5 *An open set* G *is a region iff it is polygonally connected.*

Definition 2.6 *The disk* $D(a; r)$ *is* $\{z \in \mathbf{C} : |\mathbf{z} - \mathbf{a}| < \mathbf{r}\}$*, in other words the disk with centre* a *and radius* r.

Definition 2.7 *The annulus* A *is* $\{z : r_1 < |z - a| < r_2\}$

Definition 2.8 *Two closed paths* γ_1, γ_2 *in a region* G *are* homotopic *if* γ_1 *can be deformed into* γ_2 *without leaving* G*. (Imagine these as rubber bands fixed at the ends; they are homotopic if it is possible to deform one into the other without moving the ends and without leaving* G*.)*

For example, any two closed paths in $\mathbf{C}$ are homotopic (in particular all can be shrunk to the constant path at a point).

Definition 2.9 *A region is* simply connected *if every closed path can be shrunk to a point (in other words every closed path is homotopic to the constant path).*

Definition 2.10 *A set* A *is* convex *if, whenever two points* a *and* b *are in* A*, then the line segment between* a *and* b *is also contained in* A.