Solution to First Midterm , MATC34, Oct 20, 2000

1. (a) Limit fails to exist, because if $z = re^{i\theta}$, $\bar{z}/z = e^{-2i\theta}$ and this depends on θ

   (b) This is $f(z) = \bar{z}$. The limit as $z \to 0$ is 0.

2. (this was discussed in class Oct 24 2013)

3. (a) By Cauchy's theorem, this integral is 0 because 12 is outside γ

   (b) The denominator is $(z - 3/2)(z - 1/2)$ so the function is

   $$\frac{1}{2}\left(\frac{1}{(z-3/2)} - \frac{1}{(z-1/2)}\right)$$

   The integral of the first term is 0 (since 3/2 is outside the contour of integration) and the integral of the second term is $2\pi i$ (since 1/2 is inside the contour of integration).

   (c) This integral is 0 (this was discussed in class Oct 24 2013)

4. This is true by the Deformation Theorem.

5. If there were such a function, then the integral around any contour γ would be 0, because it would be $F(b) - F(a)$ where a and b are the endpoints of the path (by the Fundamental Thm of Calculus) where $a = b$.

   $\frac{1}{z^2+1} = \frac{1}{(z-i)(z+i)} = \frac{1}{2}\left(\frac{1}{z-i} - \frac{1}{z+i}\right)$.

   We take γ to be a small circle with centre i (and radius any number < 1. Then γ contains $+i$ but not $-i$. So the integral of this function around γ is $2\pi i$.