University of Toronto at Scarborough
Division of Physical Sciences, Mathematics

MAT C34F 2002/2003

Midterm Exam

Monday, October 21, 2002; 110 minutes

No books or calculators may be used

You may use any theorems stated in class, as long as you state them clearly and correctly. This exam has four questions. Each question is worth 25 points.

1. (a) Is the function
$$f(z) = z|z|^2$$
differentiable for any z in the complex plane? If so, for which ones?
(b) Let $g(z) = \bar{z}$. Compute the limit
$$\lim_{z\to 0} g(z)$$

2. (a) Compute the values of the following integrals around the circle $\gamma(1;1)$, with centre 1 and radius 1, and with counterclockwise orientation:
$\int_{\gamma(1;1)} f(z)dz$, where
$f(z) = \frac{1}{(z-1)(z-i)}$
(b) Let γ be the square with vertices $2-2i$, $2+2i$, $-2+2i$, $-2-2i$ traversed in the counterclockwise direction. Compute
$$\int_\gamma f(z)dz$$
where
$$f(z) = \frac{1}{(z-i)(z-3i)}.$$

3. Can there be a function F with
$$\frac{dF}{dz} = \frac{1}{z}$$
for all z in the punctured disc $D'(0,1)$ with centre 0 and radius 1?

4. (a) State the Cauchy-Riemann equations.
(b) Use the Cauchy-Riemann equations to prove that if a function f is holomorphic and equal to 0 on the real axis, then $f(z) = 0$ for all z.