University of Toronto at Scarborough
Division of Physical Sciences, Mathematics

MAT C34F 2001/2002

Midterm Exam

Thursday, October 18, 2001; 110 minutes

No books or calculators may be used

You may use any theorems stated in class, as long as you state them clearly and correctly. This exam has four questions. Each question is worth 25 points.

1. (a) Obtain a series expansion for the function
$$f(z) = \frac{1+z}{1-z}$$
of the form $\sum_{n=0}^{\infty} c_n z^n$, where c_n are complex constants.
   (b) What is the radius of convergence of this series?
   (c) For what values of z do the following series specify holomorphic functions?
      i. $\sum_{n=0}^{\infty} \frac{z^n}{n!}$
      ii. $\sum_{n=0}^{\infty} z^n$

2. Compute the values of the following integrals around the unit circle $\gamma(0;1)$, with counterclockwise orientation:

   $\int_{\gamma(0;1)} f(z)dz$, where

   (a) $f(z) = \frac{1-e^z}{z}$
   (b) $f(z) = \sin(z)$
   (c) $f(z) = \frac{z}{z-2}$

3. (a) Prove that $f(z) = \bar{z}$ is not differentiable at any point of $\mathbf{C}$.
   (b) Prove that if f is holomorphic on the complex plane $\mathbf{C}$ and the real part $\mathrm{Re}(f)$ is constant then f is constant.

4. Compute the integral of the function $f(z)$
$$f(z) = \frac{1}{z^2+z+1}$$
about the following contours. All the contours should be traversed counterclockwise.

   (a) γ is a rectangle with vertices -2, 2, $-2+2i$ and $2+2i$.
   (b) γ is a rectangle with vertices -2, 2, $-2-2i$ and $2-2i$.
   (c) γ is a circle with centre 0 and radius 2.