University of Toronto at Scarborough
Division of Physical Sciences, Mathematics

MAT C34F 2000/2001

Midterm Exam

Friday, October 20, 2000; 110 minutes

No books or calculators may be used

You may use any theorems stated in class, as long as you state them clearly and correctly.

1. **(15 points)** Either obtain the limit $\lim_{z\to 0} f(z)$ or prove that the limit fails to exist:

   (a) $f(z) = \frac{\bar{z}}{z}$

   (b) $f(z) = \frac{|z|^2}{z}$

2. **(15 points)** Prove that f defined by
$$f(z) = |z|^2$$
   is differentiable only at 0, and that it is holomorphic nowhere.

3. **(30 points)** Let γ denote the contour that is the boundary of the unit disc $|z| \le 1$, oriented counterclockwise. Evaluate the following integrals:

   (a) $\int_\gamma \frac{1}{z-12} dz$

   (b) $\int_\gamma \left(\frac{1}{z^2-2z+(3/4)}\right) dz$

   (c) $\int_\gamma \frac{1}{z^2} dz$

4. **(20 points)** Let γ be the closed contour $\gamma(t) = (cost, 2\sin t)$. (This contour traces out an ellipse satisfying the equation $4x^2 + y^2 = 4$.) Show that the integral
$$\int_\gamma \frac{1}{z} dz = 2\pi i.$$

5. **(20 points)** Show that there does not exist a function F defined on all of the upper half plane $\{z \,|Im(z) \geq 0\}$ for which

$$F'(z) = \frac{1}{z^2+1}.$$

(Hint: find a contour γ in the upper half plane for which the integral

$$\int_\gamma \frac{1}{z^2+1} dz$$

is nonzero.)