University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MAT C34F 2018/19

Final

Thursday, December 20, 2018, 7:00 pm –10:00 pm

FAMILY NAME: _____

GIVEN NAMES: _____

STUDENT NUMBER: _____

SIGNATURE: _____

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO.

- There are 8 numbered pages in this exam. It is your responsibility to ensure that, at the start of the exam, this booklet has all its pages.
- No books or calculators may be used. NO cell phones or pagers are allowed at the desk or on your person during the exam.
- You may use any theorems stated in class, as long as you state them clearly and correctly.

FOR MARKERS ONLY	
Question	Marks
1	/ 10
2	/ 15
3	/ 15
4	/ 15
5	/ 15
6	/ 15
7	/ 15
TOTAL	**/100**

No books or calculators may be used

You may use any theorems stated in class, as long as you state them clearly and correctly.

(1) (10 pts) (a) State the Cauchy-Riemann equations.

(b) Let $f(u, v)$ be a complex-valued function on the complex plane. Show that if $\partial f/\partial v = 0$ for all u and v then f is constant.

(2) (15 pts) Use the Cauchy residue theorem to compute

$$\int_{|z|=2} \frac{dz}{(z-1)(z-i)^2}.$$

The line integral is around a circle of radius 2 and center 0 in the complex plane.

(3) (15 pts)

(a) (8 points) Find the Laurent series of $\frac{1}{(z+1)^2}$ around 0. What it its radius of convergence

(b) (7 pts) Find the Laurent series of $\frac{1}{z+1}$ around -1. What is its radius of convergence?

(4) (15 pts)

Compute the integral

$$\int_\gamma z^n(1-z)^m dz$$

where m is a nonnegative integer and n is an i integer. The curve γ is a circle of radius 2 and center 0 in the complex plane.

(5) (15 pts) (a) Use the Cauchy integral formula to compute the integral

$$\int_\gamma \frac{z^3+5}{z-i} dz$$

Here γ is a circle of radius 2 and center 0 in the complex plane.

(6) (15 pts)

(a) Find the singularities of $\frac{\cos(z)}{\sin(z)}$. State the type of singularity (removable singularity, pole, essential singularity).

(b) Compute the residue of $\frac{\cos(z)}{\sin(z)}$ at $z = 0$.

(7) (15 points)

Use residues to compute the integral

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+9)}.$$

(This blank page for question 2)