MAT C34F   2013/14

<u>Problem Set #6</u>

Due date: Thursday, November 28, 2013 at the beginning of class

Do the following problems.

1. Find the images of (a) $\{z : 0 < \arg(z) < \pi/6\}$ (b) $D(0; 2)$ (c) $\{z : 0 < \text{Im}(z) < 1\}$ under $z \mapsto 1/z$.

2. Describe the image of $\{z : 0 < \arg(z) < \pi/2\}$ under $z \mapsto w = \frac{z-1}{z+1}$

3. Describe the image of $\{z : \text{Re}(z) > 0\}$ under $z \mapsto w$ where $\frac{w-1}{w+1} = 2\frac{z-1}{z+1}$

4. Find Möbius transformations to map

   (i) $1, i, 0$ to $1, i, -1$ respectively

   (ii) $0, 1, \infty$ to $\infty, -i, 1$ respectively

5. Find the Möbius transformation mapping $0, 1, \infty$ to $1, 1+i, i$ respectively. Under this mapping what is the image of a circular arc through -1 and $-i$?

6. Describe the Möbius transformations mapping the open upper half plane onto $D(0; 1)$ which map the imaginary axis onto the real axis.

7. Find the image of (i) $\{z : 0 < \text{Arg}(z) < \pi/4\}$ under $z \mapsto iz^4$

   (ii) Find the image of $\{z : 0 < \text{Re}(z) < 1, 0 < \text{Im}(z) < \pi/2\}$ under $z \mapsto e^z$

8. Construct a conformal map onto $D(0; 1)$ for $\{z : 1- < \text{Re}(z) < 1\}$

9. Check that each of the following functions is harmonic on the indicated set, and find a holomorphic function of which it is the real part.

   (i) $\sin(x^2 - y^2)e^{-2xy}$

   (ii) $\log(x^2 + y^2)^{3/2}$ (on the open first quadrant).