University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MAT C34F 2018/19

Problem Set #1

Due date: Thursday, September 20, 2018 at the beginning of class

REVISED VERSION September 13, 2018. The only change is to the reading list. The assignment questions have not changed.

Reading: Priestley Chap. 1, and Chap 3, Chap. 5 and Chap. 6

Solve the following problems.

(1) Prove that the function $\exp(\bar{z})$ is not holomorphic anywhere.
(2) Find all the roots of the equation $\sin(z) = \cosh(4)$ by equating the real and imaginary parts of $\sin(z)$ and $\cosh(4)$.
(3) Express each of the following in polar coordinates: i, $1-i$, $\sqrt{3}-i$
(4) Express each of the following as $x+iy$: $e^{4\pi i/3}, e^{5\pi i/6}, (1+i)^{-3}$
(5) Describe each of the following sets geometrically. Which are open, which are closed, and which are compact? (i) $\{z : |z-1-i| = 1\}$ (ii) $\{z : |z-1+i| \geq |z-1-i|\}$ (iii) $\{z : |z+i| \neq |z-i|\}$ (iv) $\{z = |z|e^{i\theta} : \pi/4 < \theta < 3\pi/4\}$
(6) For each of the following choices of f, either obtain $\lim_{z\to 0} f(z)$ or prove that the limit does not exist. (i) $|z|^2/z$,
 (ii) $\bar{z}/z$
(7) Prove that f is continuous on $\mathbb{C}$ when (i) $f(z) = \bar{z}$
 (ii) $f(z) = \mathrm{Im}(z)$
 (iii) $f(z) = \mathrm{Re}(z^3)$
(8) Prove that f defined by $f(z) = z^5/|z|^4$ $(z \neq 0)$, $f(0) = 0$ satisfies the Cauchy-Riemann equations at $z = 0$ but is not differentiable there.
(9) Which of the following are holomorphic?
 (i) $e^z/z(z-1)(z-2)$
 (ii) $(1+e^z)^{-1}$
(10) Where do the following series define holomorphic functions?
 (i) $\sum_{n=1}^{\infty}(-1)^n z^n/n!$
 (ii) $\sum_{n=0}^{\infty} z^{5n}$
(11) Determine for which values of z the following series converge absolutely:
 (i) $\sum_{n=0}^{\infty} \frac{(z+1)^n}{2^n}$
 (ii) $\sum_{n=0}^{\infty} \left(\frac{z-1}{z+1}\right)^n$
(12) Write down an expansion of the form $\sum_{n=0}^{\infty} c_n z^n$ for
 (i) $\frac{1}{1+z^4}$
 (ii) $\frac{1}{1+z+z^2}$
(13) Find all solutions of $\cos^2 z = 4$.