

SOLUTIONS ASSIGNMENT #5 MATA33

$$\textcircled{1} \quad A = \begin{bmatrix} 1 & 2 \\ 4 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -4 \\ 7 & 1 \end{bmatrix}$$

$$(a) \det(A) = -2 - 8 = -10 \quad \det(B) = 3 + 28 = 31$$

$$AB = \begin{bmatrix} 1 & 2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 7 & 1 \end{bmatrix} = \begin{bmatrix} 17 & -2 \\ -2 & -18 \end{bmatrix}$$

$$\det(AB) = -310 = (-10)(31) = \det(A)\det(B).$$

$$(b) A + B = \begin{bmatrix} 4 & -2 \\ 11 & -1 \end{bmatrix} \quad \det(A+B) = -4 + 22 = 18$$

$$\det(A) + \det(B) = -10 + 31 = 21$$

$$\therefore \det(A+B) \neq \det(A) + \det(B)$$

$$(c) A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -2 & -2 \\ -4 & 1 \end{bmatrix} = -\frac{1}{10} \begin{bmatrix} -2 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{2}{5} & -\frac{1}{10} \end{bmatrix}$$

$$\therefore \det(A^{-1}) = -\frac{1}{50} - \frac{2}{25} = -\frac{5}{50} = -\frac{1}{10} = \frac{1}{\det(A)}$$

(You can also calculate as follows:

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -2 & -2 \\ -4 & 1 \end{bmatrix} \text{ so } \det(A^{-1}) = \frac{1}{(\det(A))^2} (-2 - 8)$$

$$= \frac{-10}{(-10)^2} = -\frac{1}{10}$$

$$= \frac{1}{\det(A)}$$

$$(d) C = \begin{bmatrix} p & 2p \\ 4q & -2q \end{bmatrix} \quad \det(C) = -2pq - 8pq \\ = pq(-10) = pq \det(A) \quad (2)$$

(e) For $x \in \mathbb{R}$, the 2×2 matrix $xI - A$ is invertible if and only if $|xI - A| \neq 0$ (i.e. $\det(xI - A) \neq 0$)

Consider solving $|xI - A| = 0$.

$$|xI - A| = \left| \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 4 & -2 \end{bmatrix} \right| = \left| \begin{bmatrix} x-1 & -2 \\ -4 & x+2 \end{bmatrix} \right| \\ = (x-1)(x+2) - 8 = x^2 + x - 10$$

Solve: $x^2 + x - 10 = 0$ Quadratic formula $\Rightarrow x = \frac{-1 \pm \sqrt{41}}{2}$

$$\therefore |xI - A| = 0 \text{ iff } x = \frac{-1 \pm \sqrt{41}}{2}$$

thus, $xI - A$ is invertible iff $x \neq \frac{-1 \pm \sqrt{41}}{2}$

"iff" = "if and only if"

$$(2) A = \begin{bmatrix} 1 & 3 & -2 \\ -2 & 1 & -1 \\ 0 & 4 & 3 \end{bmatrix} + B = \begin{bmatrix} 0 & -2 & 3 \\ -2 & 4 & -2 \\ 3 & 1 & -1 \end{bmatrix}$$

(a) We find $\det(A)$ by expanding along row 3 (to take advantage of the 0 in entry a_{31})

$$\det(A) = (4)(-1) \begin{vmatrix} 1 & -2 \\ -2 & -1 \end{vmatrix} + (3)(1) \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = (-4)(-5) + (3)(7) \\ = 41$$

$$\det(B) = (-2)(-1) \begin{vmatrix} -2 & -2 \\ 3 & -1 \end{vmatrix} + (3) \begin{vmatrix} -2 & 4 \\ 3 & 1 \end{vmatrix} = (2)(8) + (3)(-14) \\ = -26$$

(Expand on Row 1)

$$AB = \begin{bmatrix} 1 & 3 & -2 \\ -2 & 1 & -1 \\ 0 & 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & -2 & 3 \\ -2 & 4 & -2 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -12 & 8 & -1 \\ -5 & 7 & -7 \\ 1 & 19 & -11 \end{bmatrix} \quad (3)$$

$$\therefore \det(AB) = (8)(-1) \begin{vmatrix} -5 & -7 \\ 1 & -11 \end{vmatrix} + (7) \begin{vmatrix} -12 & -1 \\ 1 & -11 \end{vmatrix} + (19)(-1) \begin{vmatrix} -12 & -1 \\ -5 & -7 \end{vmatrix}$$

(Expand along
2nd Column) $= (-8)(62) + (7)(133) - (19)(79) = -1066$

$$\det(A)\det(B) = (41)(-26) = -1066$$

$$\therefore \det(AB) = \det(A)\det(B).$$

$$(b) \text{ Let } k \in \mathbb{R} \quad |kA| = \left| \begin{bmatrix} k & 3k & -2k \\ -2k & k & -k \\ 0 & 4k & 3k \end{bmatrix} \right|$$

$= k \begin{vmatrix} k & -k \\ 4k & 3k \end{vmatrix} + 2k \begin{vmatrix} 3k & -2k \\ 4k & 3k \end{vmatrix}$

(Expand along
column 1) $= k(7k^2) + 2k(9k^2 + 8k^2) = 7k^3 + 34k^3$
 $= 41k^3 = |k^3| |A|$

$$(c) D = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 4 & 3 \\ -2 & 1 & -1 \end{bmatrix}$$

$$\det(D) = -(3) \begin{vmatrix} 0 & 3 \\ -2 & -1 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ -2 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix}$$

(Expand along
Column 2) $= (-3)(6) + 4(-5) - 1(3)$
 $= -41 = -\det(A)$



(4)

(3) We find $|M|$ where $M = \begin{bmatrix} 2 & 0 & 3 & 1 \\ 1 & 4 & 2 & 2 \\ -1 & 3 & 1 & 4 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

Solution: It is efficient to expand along row 4 so as to take advantage of the two 0's there.

$$\begin{aligned}
 |M| &= 2 \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -1 & 1 & 4 \end{vmatrix} - 1 \begin{vmatrix} 2 & 0 & 1 \\ 1 & 4 & 2 \\ -1 & 3 & 4 \end{vmatrix} \\
 &= 2 \left(2 \begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} - 3 \begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} \right) \quad \text{(3x3 det found by expansion along row 1)} \\
 &\quad - 1 \left(2 \begin{vmatrix} 4 & 2 \\ 3 & 4 \end{vmatrix} + 1 \begin{vmatrix} 1 & 4 \\ -1 & 3 \end{vmatrix} \right) \\
 &= 2 (2(6) - 3(6) + 1(3)) - 1(2(10) + 1(7)) \\
 &= 2 (12 - 18 + 3) - 1 (20 + 7) = -33
 \end{aligned}$$

(4)(a) To find a 3×3 non-zero matrix A such that $A^2 \neq 0$, yet $A^3 = 0$, we place 1's in the top right corner and 0's every where else.

\therefore consider the upper triangular matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{We have } A \neq 0.$$

$$A^2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq 0.$$

(Note how the 1's have "drifted" to the top-right of A^2) (5)

$$\therefore A^3 = AA^2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$(A, A^2 \neq 0 \text{ yet } A^3 = 0) = 0$

(b) Assume B is an $n \times n$ matrix such that $B^k = 0$ where k is a natural number.

$$0 = \det(B^k) = (\det(B))^k \Rightarrow \det(B) = 0$$

$\therefore B$ is not invertible. □

⑤ P and Q are 5×5 matrices such that $\det(P) = 2$ and $\det(Q) = -3$

$$(a) \det(PQ^2) = \det(P) \cdot (\det(Q))^2 = (2)(-3)^2 = 18$$

$$(b) \det(3Q) = 3^5 \cdot \det(Q) = 3^5(-3) = -(3^6) \\ (\text{3 for each row}) \quad = -729 \\ (\dots 5 \text{ rows})$$

$$(c) \det(-2(P^{-1})) = (-2)^5 \det(P^{-1}) = \frac{-32}{\det(P)} = -16 \\ (-2 \text{ per row})$$

$$(d) \det(3P^{-1}) = \det\left(\frac{1}{3}(P^{-1})\right) = \left(\frac{1}{3}\right)^5 \det(P^{-1}) \\ \left(\frac{1}{3} \text{ per row}\right) \quad = \frac{1}{3^5} \cdot \frac{1}{\det(P)} \\ (\dots 5 \text{ rows}) \quad = \frac{1}{486}$$



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⑥ Problems using Cramer's rule

Recall: If A is $n \times n$ and invertible then Cramer's rule is used to solve the matrix equation (or linear system) $AX = B$

where $X = [x_i]_{i=1, \dots, n}$

$$\text{We have } x_i = \frac{\det(A_i(B))}{\det(A)}$$

where $A_i(B)$ is the $n \times n$ matrix obtained by replacing (or exchanging) the i th column of A with B .

(a) Given system is $2x - 7y = 50$
 $x + 3y = 10$

$$A = \begin{bmatrix} 2 & -7 \\ 1 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 50 \\ 10 \end{bmatrix}$$

By Cramer's rule we have

$$x = \frac{\det(A_1(B))}{\det(A)} = \frac{\begin{vmatrix} 50 & -7 \\ 10 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -7 \\ 1 & 3 \end{vmatrix}} = \frac{220}{13}$$

$$y = \frac{\det(A_2(B))}{\det(A)} = \frac{\begin{vmatrix} 2 & 50 \\ 1 & 10 \end{vmatrix}}{\begin{vmatrix} 2 & -7 \\ 1 & 3 \end{vmatrix}} = -\frac{30}{13}$$

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$$(b) \text{ Given system is } \left. \begin{array}{l} x - y - 3z = -5 \\ 2x - y - 4z = -8 \\ x + y - z = -1 \end{array} \right\} (*)$$

$$\text{Coefficient matrix is } A = \begin{bmatrix} 1 & -1 & -3 \\ 2 & -1 & -4 \\ 1 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} -5 \\ -8 \\ -1 \end{bmatrix}$$

$$x = \frac{\det(A_1(B))}{\det(A)} = \frac{\begin{vmatrix} -5 & -1 & -3 \\ -8 & -1 & -4 \\ -1 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & -3 \\ 2 & -1 & -4 \\ 1 & 1 & -1 \end{vmatrix}} \quad (\text{Expand along row 3})$$

$$= \frac{-1 \begin{vmatrix} -1 & -3 \\ -1 & -4 \end{vmatrix} \begin{vmatrix} -5 & -3 \\ -8 & -4 \end{vmatrix} \begin{vmatrix} -5 & -1 \\ -8 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & -4 \\ 2 & -1 & -4 \\ 1 & 1 & -1 \end{vmatrix}} \quad (\text{Expand along column 1})$$

$$= \frac{-1(4-3) - 1(20-24) - 1(5-8)}{(1+4) - 2(1+3) + 1(4-3)} = \frac{6}{-2} = -3$$

$$y = \frac{\det(A_2(B))}{\det(A)} = \frac{\begin{vmatrix} 1 & -5 & -3 \\ 2 & -8 & -4 \\ 1 & -1 & -1 \end{vmatrix}}{-2} \quad (\text{Expand along column 1})$$

$$= \frac{1 \begin{vmatrix} -8 & -4 \\ -1 & -1 \end{vmatrix} - 2 \begin{vmatrix} -5 & -3 \\ -1 & -1 \end{vmatrix} + 1 \begin{vmatrix} -5 & -3 \\ -8 & -4 \end{vmatrix}}{-2}$$

$$= \frac{(8-4) - 2(5-3) + 1(20-24)}{-2} = 2$$

$$z = \frac{\det(A_3(B))}{\det(A)} = \frac{\begin{vmatrix} 1 & -1 & -5 \\ 2 & -1 & -8 \\ 1 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & -8 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} - 2 \begin{vmatrix} 1 & -1 & -5 \\ 1 & -1 & -1 \end{vmatrix} + 1 \begin{vmatrix} -1 & -5 \\ -1 & -8 \end{vmatrix}}$$

$$= \frac{(1+8) - 2(1+5) + 1(8-5)}{-2} = \frac{0}{-2} = 0$$

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∴ the solution to (*) is $x = -3, y = 2, z = 0$.



(7) Compact cars cost \$c/car
Sedans cost \$d/car, $d > c$

Budget is \$m and a total of n cars are purchased

(a) x = number of compact cars purchased

y = " " sedans purchased

System of equations is $\begin{cases} x+y=n \\ cx+dy=m \end{cases}$ (*)

The matrix equation corresponding to (*) is

$A\bar{X} = B$ where $A = \begin{bmatrix} 1 & 1 \\ c & d \end{bmatrix}, \bar{X} = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} n \\ m \end{bmatrix}$

(b) When we have A^{-1} , then the solution to (*) is $\bar{X} = A^{-1}B$ (Text reference is page 279)

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The inverse of A obtained by the "determinant formula" is

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -1 \\ -c & 1 \end{bmatrix} = \frac{1}{d-c} \begin{bmatrix} d & -1 \\ -c & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{d-c} \begin{bmatrix} d & -1 \\ -c & 1 \end{bmatrix} \begin{bmatrix} n \\ m \end{bmatrix} = \begin{bmatrix} \frac{dn-m}{d-c} \\ \frac{m-nc}{d-c} \end{bmatrix}$$

Thus $x = \frac{dn-m}{d-c}$, $y = \frac{m-nc}{d-c}$ (Note that $\det(A) \neq 0$ since $d > c$)

(c) Now we use the method of reduction to find $x \circ y$ (i.e to solve $(*)$)

$$\left[\begin{array}{cc|c} 1 & 1 & n \\ c & d & m \end{array} \right] R_2 - cR_1 \rightarrow R_2 \left[\begin{array}{cc|c} 1 & 1 & n \\ 0 & d-c & m-cn \end{array} \right]$$

$$\left(\frac{1}{d-c} \right) R_2 \rightarrow R_2 \left[\begin{array}{cc|c} 1 & 1 & n \\ 0 & 1 & \frac{m-cn}{d-c} \end{array} \right]$$

$$R_1 - R_2 \rightarrow R_1 \left[\begin{array}{cc|c} 1 & 0 & \frac{nd-m}{d-c} \\ 0 & 1 & \frac{m-cn}{d-c} \end{array} \right]$$

$$\therefore x = \frac{nd-m}{d-c} \quad \text{and} \quad y = \frac{m-cn}{d-c}$$

(d) Next we use Cramer's rule to solve $(*)$. This approach is valid because $d > c$ implies

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that $\det(A) \neq 0$.

$$x = \frac{\det(A_1(B))}{\det(A)} = \frac{\det \begin{bmatrix} n & 1 \\ m & d \end{bmatrix}}{d-c} = \frac{nd-m}{d-c}$$

$$y = \frac{\det(A_2(B))}{\det(A)} = \frac{\det \begin{bmatrix} 1 & n \\ c & m \end{bmatrix}}{d-c} = \frac{m-cn}{d-c}$$

(e) We verify our calculations and results in (b), (c), (d)
by substituting into (*):

$$x+y = \frac{nd-m}{d-c} + \frac{m-cn}{d-c} = \frac{nd-cn}{d-c} = n \quad \checkmark$$

$$\begin{aligned} cx+dy &= c\left(\frac{nd-m}{d-c}\right) + d\left(\frac{m-cn}{d-c}\right) \\ &= \frac{cnd-cm+dm-cdn}{d-c} = m \quad \checkmark \end{aligned}$$

That shows $x = \frac{nd-m}{d-c}$ & $y = \frac{m-cn}{d-c}$ $\underline{\underline{do}}$ satisfy (*).

(f) To begin, recall that $d > c$ so $d-c > 0$.
We certainly want $x \geq 0$ & $y \geq 0$.

Using $x = \frac{nd-m}{d-c}$, we want $nd-m > 0$ so $m \leq nd$.

Also, $y = \frac{m-cn}{d-c}$ so we want $m-cn > 0$, hence
 $m > cn$.

Combine these last two inequalities to get
 $cn \leq m \leq nd$. Our desired inequality is $c \leq \frac{m}{n} \leq d$.

(g) Let's assume that $c \leq \frac{m}{n} \leq d$. (as in part(f)) (11)

We still have $\frac{nd-m}{d-c} = x$ and $y = \frac{m-cn}{d-c}$

But it may not be the case (actually, it probably isn't)

that $\frac{nd-m}{d-c}$ and $\frac{m-cn}{d-c}$ are non-negative

integers. (Remember: x & y are numbers of cars purchased.) Here is a "quasi-realistic" example. Suppose $c = 22$ (000's... k) $n = 48$
 $d = 37$ k $m = 1,680k$

$$\therefore x = \frac{nd-m}{d-c} = \frac{96}{15} = 6.4 \dots \text{not an integer}$$

$$y = \frac{m-cn}{d-c} = \frac{624}{15} = 41.6 \dots \text{not an integer}$$



⑧ Refer to the linear system on page 273, Example 1.

The asserted solution is:

$$x_1 = 1 - r - 3s$$

$$x_2 = -2 - 2r - s$$

$$x_3 = r$$

$$x_4 = s$$

$r, s \in \mathbb{R}$ are parameters
(\therefore can take on any real values)

Substitute into each equation in the system:

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$$\underline{\text{Eq}^{\Delta} 1:} \quad x_1 + 2x_2 + 5x_3 + 5x_4$$

$$= 1 - r - 3s + 2(-2 - 2r - s) + 5r + 5s$$

$$= -3 - r - 3s - 4r - 2s + 5r + 5s$$

$$= -3 \quad \checkmark$$

$$\underline{\text{Eq}^{\Delta} 2:} \quad x_1 + x_2 + 3x_3 + 4x_4$$

$$= 1 - r - 3s + (-2 - 2r - s) + 3r + 4s$$

$$= 1 - r - 3s - 2 - 2r - s + 3r + 4s$$

$$= -1 \quad \checkmark$$

$$\underline{\text{Eq}^{\Delta} 3:} \quad x_1 - x_2 - x_3 + 2x_4$$

$$= 1 - r - 3s - (-2 - 2r - s) - r + 2s$$

$$= 1 - r - 3s + 2 + 2r + s - r + 2s$$

$$= 3 \quad \checkmark$$

That completes the verification. 

$$\textcircled{9} \quad \text{Given system is } \begin{cases} 3sx - 2y = 4 \\ -6x + sy = 1 \end{cases} \quad \{ (\ast)$$

$s \in \mathbb{R}$ is a parameter ; x, y are variables .

$$\text{Coefficient matrix is } A = \begin{bmatrix} 3s & -2 \\ -6 & s \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

(\ast) has a unique solution if and only if

$$\det(A) \neq 0 \quad (\text{i.e. } 3s^2 - 12 \neq 0)$$

\therefore we have a unique sol \cong for all $s \in \mathbb{R}, s \neq \pm 2$.

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Using Cramer's rule :

$$x = \frac{\det(A_1(B))}{\det(A)} = \frac{\det \begin{bmatrix} 4 & -2 \\ 1 & s \end{bmatrix}}{3s^2 - 12} = \frac{4s + 2}{3s^2 - 12}$$

$$y = \frac{\det(A_2(B))}{\det(A)} = \frac{\det \begin{bmatrix} 3s & 4 \\ -6 & 1 \end{bmatrix}}{3s^2 - 12} = \frac{3s + 24}{3s^2 - 12}$$

$$= \frac{s + 8}{s^2 - 4}$$

(x, y are solutions to $(*)$ for all $s \in \mathbb{R}, s \neq \pm 2$)



⑩ Carefully read problem 13 on page 284.

We have three companies D, E, F

Let x, y, z be the number of shares bought for company D, E, F, respectively.

We summarize data about D, E, F shares in the following table :

	D	E	F
% annual growth	16%	12%	9%

Share price \$60	\$80	\$30
per share		

A total of \$500,000 is invested.

We have two extra conditions :

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- (i) 4 times as much F is purchased as E
- (ii) combined growth (i.e. growth of entire portfolio)
is 13.68%

(a) Now we develop our equations.

$$60x + 80y + 30z = 500,000$$

Divide through

by 10 to get $6x + 8y + 3z = 50,000 \quad (1)$

"Combined growth
equation"

$$\begin{aligned} (.16)(60x) + (.12)(80y) + (.09)(30z) \\ = (.1368)(60x + 80y + 30z) \end{aligned}$$

Simplify: $1.392x - 1.344y - 1.404z = 0$

$$\therefore 1392x - 1344y - 1404z = 0$$

$$\therefore 116x - 112y - 117z = 0 \quad (2)$$

$$-4y + z = 0 \quad (3) \quad (\text{i.e. "describing"})$$

\therefore the system of linear equations "governing" the investment is:

$$6x + 8y + 3z = 50,000$$

$$116x - 112y - 117z = 0$$

$$0x - 4y + z = 0$$

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(b) The matrix equation for this system is

$A \mathbf{x} = \mathbf{B}$ which is

$$\underbrace{\begin{bmatrix} 6 & 8 & 3 \\ 116 & -112 & -117 \\ 0 & -4 & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_X = \underbrace{\begin{bmatrix} 50,000 \\ 0 \\ 0 \end{bmatrix}}_B$$

We let $A_i(B)$ be the matrix (3×3) obtained by replacing the i^{th} column of A with B

$$\text{Then } x = \frac{\det(A_1(B))}{\det(A)}, y = \frac{\det(A_2(B))}{\det(A)}$$

$$\text{and } z = \frac{\det(A_3(B))}{\det(A)}$$

$$\det(A) = 4 \begin{vmatrix} 6 & 3 \\ 116 & -117 \end{vmatrix} + 1 \begin{vmatrix} 6 & 8 \\ 116 & -112 \end{vmatrix} = 4(-1050) + (-1600) = -5800$$

$$\det(A_1(B)) = \begin{vmatrix} 50,000 & 8 & 3 \\ 0 & -112 & -117 \\ 0 & -4 & 1 \end{vmatrix} = 50,000 \begin{vmatrix} -112 & -117 \\ -4 & 1 \end{vmatrix} = 50,000(-112 - 468) = -29,000,000$$

$$\det(A_2(B)) = \begin{vmatrix} 6 & 50,000 & 3 \\ 116 & 0 & -117 \\ 0 & 0 & 1 \end{vmatrix} = 50,000(116) = -5,800,000$$

$$\det(A_3(B)) = \begin{vmatrix} 6 & 8 & 50,000 \\ 116 & -112 & 0 \\ 0 & -4 & 0 \end{vmatrix} = 50,000(-464) = -23,200,000$$

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$$\text{We now have } x = \frac{-29,000,000}{-5,800} = 5,000$$

$$y = \frac{-5,800,000}{-5,800} = 1,000$$

$$z = \frac{-23,200,000}{-5,800} = 4,000$$



- (11) Let A be an $n \times n$ matrix such that the entries in each row add up to 0.

We prove $\det(A) = 0$.

Solution: Suppose $\det(A) \neq 0$. We then know that A is invertible, so the unique solution to the homogeneous system of equations

$$AX = 0 \text{ is } X = A^{-1}0 = 0. \quad N = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

Since $AN = 0$ where N is the $n+1$ column matrix consisting of 1's, N is also a solution to the system $AX = 0$, and $N \neq 0$. This contradicts the uniqueness of the solution to $AX = 0$ when A is invertible. Thus, A cannot be invertible so $\det(A) = 0$.

BIG END

