

Solutions to MATA33 ASSIGNMENT 3

① Section 6.4 Problems

#2 $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ is reduced.

#4 $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ is not reduced.

Reason: the (1,1) entry should = 0,
not 1. Entries above & below a leading 1 must = 0.

#6 $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is not reduced

Reason: the leading

1 in row 2 is to
the left of the leading 1
in row 1.

#8 $\begin{bmatrix} 0 & -3 & 0 & 2 \\ 1 & 5 & 0 & 2 \end{bmatrix} R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 5 & 0 & 2 \\ 0 & -3 & 0 & 2 \end{bmatrix}$

$(-\frac{1}{3})R_2 \rightarrow R_2 \begin{bmatrix} 1 & 5 & 0 & 2 \\ 0 & 1 & 0 & -\frac{2}{3} \end{bmatrix} R_1 - 5R_2 \rightarrow R_1 \begin{bmatrix} 1 & 0 & 0 & \frac{16}{3} \\ 0 & 1 & 0 & -\frac{2}{3} \end{bmatrix}$

(Reduced matrix)

#10 $\begin{bmatrix} 2 & 3 \\ 1 & -6 \\ 4 & 8 \\ 1 & 7 \end{bmatrix} R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & -6 \\ 2 & 3 \\ 4 & 8 \\ 1 & 7 \end{bmatrix} R_2 - 2R_1 \rightarrow R_2 \begin{bmatrix} 1 & -6 \\ 0 & 15 \\ 4 & 8 \\ 1 & 7 \end{bmatrix} R_3 - 4R_1 \rightarrow R_3 \begin{bmatrix} 1 & -6 \\ 0 & 15 \\ 0 & 32 \\ 1 & 7 \end{bmatrix} R_4 - R_1 \rightarrow R_4 \begin{bmatrix} 1 & -6 \\ 0 & 15 \\ 0 & 32 \\ 0 & 13 \end{bmatrix}$

$(\frac{1}{15})R_2 \rightarrow R_2 \begin{bmatrix} 1 & -6 \\ 0 & 1 \\ 0 & 32 \\ 0 & 13 \end{bmatrix} R_1 + 6R_2 \rightarrow R_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 32 \\ 0 & 13 \end{bmatrix} R_3 - 32R_2 \rightarrow R_3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 13 \end{bmatrix} R_4 - 13R_2 \rightarrow R_4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ (Reduced Matrix)

#12 $\begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 3 \\ 0 & -1 & 0 \\ 0 & 4 & 1 \end{bmatrix} R_1 \leftrightarrow R_2 \begin{bmatrix} 2 & 0 & 3 \\ 0 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 4 & 1 \end{bmatrix} (\frac{1}{2})R_1 \rightarrow R_1 \begin{bmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

(2)

$$\begin{array}{l}
 R_2 \leftrightarrow R_3 \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & \\ 0 & -1 & 0 & \\ 0 & 0 & 2 & \\ 0 & 4 & 1 & \end{array} \right] (-1)R_2 \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & \\ 0 & 1 & 0 & \\ 0 & 0 & 2 & \\ 0 & 4 & 1 & \end{array} \right] \\
 R_4 - 4R_2 \rightarrow R_4 \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & \\ 0 & 1 & 0 & \\ 0 & 0 & 2 & \\ 0 & 0 & 1 & \end{array} \right] \left(\frac{1}{2} \right) R_3 \rightarrow R_3 \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 1 & \end{array} \right] \\
 R_1 - \frac{3}{2}R_3 \rightarrow R_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \end{array} \right] (\text{Reduced Matrix}) \quad \blacksquare
 \end{array}$$

#14 Augmented matrix followed by reduction:

$$\begin{array}{l}
 \left[\begin{array}{cc|c} 1 & -3 & -11 \\ 4 & 3 & 9 \end{array} \right] R_2 - 4R_1 \rightarrow R_1 \left[\begin{array}{cc|c} 1 & -3 & -11 \\ 0 & 15 & 53 \end{array} \right] \\
 \left(\frac{1}{15} \right) R_2 \rightarrow R_2 \left[\begin{array}{cc|c} 1 & -3 & -11 \\ 0 & 1 & \frac{53}{15} \end{array} \right] R_1 + 3R_2 \rightarrow R_1 \left[\begin{array}{cc|c} 1 & 0 & -\frac{2}{5} \\ 0 & 1 & \frac{53}{15} \end{array} \right] \\
 \therefore \text{solution is } x = -\frac{2}{5} \text{ and } y = \frac{53}{15} \quad \text{(Reduced)} \quad \blacksquare
 \end{array}$$

#16 Augmented matrix followed by reduction is:

$$\begin{array}{l}
 \left[\begin{array}{ccc|c} 3 & 2 & -1 & 1 \\ -1 & -2 & -3 & 1 \end{array} \right] R_1 \leftrightarrow R_2 \left[\begin{array}{ccc|c} -1 & -2 & -3 & 1 \\ 3 & 2 & -1 & 1 \end{array} \right] (-1)R_1 \rightarrow R_1 \left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 3 & 2 & -1 & 1 \end{array} \right] \\
 R_2 - 3R_1 \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & -4 & -10 & 4 \end{array} \right] \left(-\frac{1}{4} \right) R_2 \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 1 & \frac{5}{2} & -1 \end{array} \right] \left\{ \begin{array}{l} \text{It's useful.} \\ \text{to verify} \\ \text{by substitution} \end{array} \right. \\
 R_1 - 2R_2 \rightarrow R_1 \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{5}{2} & -1 \end{array} \right] \left(\text{Reduced matrix} \right) \\
 \therefore x - 2z = 1 \quad \therefore \text{solution is } x = 2r+1, y = -\frac{5}{2}r-1, z = r \\
 y + \frac{5}{2}z = -1 \quad \text{where } r \in \mathbb{R} \text{ is arbitrary.} \quad \blacksquare
 \end{array}$$

(3)

#18 Augmented matrix followed by reduction:

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 1 & 1 & 5 & 10 \end{array} \right] R_2 - R_1 \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -2 & 3 & 9 \end{array} \right]$$

$$(-\frac{1}{2})R_2 \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & -\frac{3}{2} & -\frac{9}{2} \end{array} \right] R_1 - 3R_2 \rightarrow R_1 \quad \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & \frac{13}{2} \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & 0 & \frac{13}{2} \end{array} \right] \quad \left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{2} & -\frac{9}{2} \end{array} \right]$$

"Basic Variables" } Isolate those variables that have columns containing leading 1's. They are x & y .

We write them in terms of z

(which has no leading 1 in its column)

$$\text{Row 1} \Rightarrow x = \frac{29}{2} - \frac{13}{2}z \quad \left. \begin{array}{l} \\ \end{array} \right\} z \text{ is a "free variable".}$$

$$\text{Row 2} \Rightarrow y = -\frac{9}{2} + \frac{3}{2}z \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{We let } z=r$$

$$\therefore \text{solution is } x = -\frac{13}{2}r + \frac{29}{2}$$

$$(\text{i.e. solution to the given system}) \quad y = \frac{3}{2}r - \frac{9}{2}$$

$$z = r \quad \text{where } r \in \mathbb{R}.$$



#20 Augmented matrix followed by reduction:

$$\left[\begin{array}{cc|c} 1 & 4 & 9 \\ 3 & -1 & 6 \\ 1 & -1 & 2 \end{array} \right] R_2 - 3R_1 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & 4 & 9 \\ 0 & -13 & -21 \\ 0 & -5 & -7 \end{array} \right]$$

$$(-\frac{1}{13})R_2 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & 4 & 9 \\ 0 & 1 & \frac{21}{13} \\ 0 & -5 & -7 \end{array} \right] R_3 - 5R_2 \rightarrow R_3 \quad \left[\begin{array}{cc|c} 1 & 4 & 9 \\ 0 & 1 & \frac{21}{13} \\ 0 & 0 & \frac{14}{13} \end{array} \right]$$

Last row gives the equation $0x_1 + 0x_2 = \frac{14}{13}$, for which there is no solution. \therefore original system has no solution.



(4)

#22 Augmented matrix followed by reduction:

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 2 & -3 & -2 & 4 \\ 1 & -1 & -5 & 23 \end{array} \right] R_2 - 2R_1 \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & -5 & 0 & -10 \\ 1 & -1 & -5 & 23 \end{array} \right] R_3 - R_1 \rightarrow R_1$$

$$\left(-\frac{1}{5} \right) R_2 \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -4 & 16 \end{array} \right] R_1 - R_2 \rightarrow R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -4 & 16 \end{array} \right] R_3 + 2R_2 \rightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & 20 \end{array} \right]$$

$$\left(-\frac{1}{4} \right) R_3 \rightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -5 \end{array} \right] R_1 + R_3 \rightarrow R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -5 \end{array} \right]$$

∴ Solution to the given system
is $x = 0, y = 2, z = -5$

(Reduced Matrix)



#28 Let x represent the number of units of product A.
" " y " " " " " " B.

25% more of A can be sold than of B

∴ the amount of A that sells is 25% more
than the amount of B that sells

$$\therefore x = y + .25y = \frac{5}{4}y = 1.25y$$

For profit, $8x + 11y = 42,000$

$$\text{We have the system: } x - \frac{5}{4}y = 0 \\ 8x + 11y = 42,000$$

$$\left[\begin{array}{cc|c} 1 & -\frac{5}{4} & 0 \\ 8 & 11 & 42,000 \end{array} \right] R_2 - 8R_1 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & -\frac{5}{4} & 0 \\ 0 & 21 & 42,000 \end{array} \right]$$

$$\left(\frac{1}{21} \right) R_2 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & -\frac{5}{4} & 0 \\ 0 & 1 & 2000 \end{array} \right] R_1 + \frac{5}{4}R_2 \rightarrow R_1 \quad \left[\begin{array}{cc|c} 1 & 0 & 2500 \\ 0 & 1 & 2000 \end{array} \right]$$

∴ sell 2,500 units of A + 2,000 units of B.



(5)

#30 Let x represent the number of desks produced at the East Coast plant and let y be the number of desks produced at the West Coast plant.

\therefore total cost for production at the plants is to be the same, we have the "cost eq"

$$90x + 20,000 = 95y + 18,000$$

Total production is to be 800 thus

$$x + y = 800$$

\therefore our system is :
$$\begin{aligned} x + y &= 800 \\ 90x - 95y &= -2,000 \end{aligned}$$

Form augmented matrix and reduce :

$$\left[\begin{array}{cc|c} 1 & 1 & 800 \\ 90 & -95 & -2000 \end{array} \right] R_2 - 90R_1 \rightarrow R_2 \left[\begin{array}{cc|c} 1 & 1 & 800 \\ 0 & -185 & -74,000 \end{array} \right]$$

$$\left(-\frac{1}{185} \right) R_2 \rightarrow R_2 \left[\begin{array}{cc|c} 1 & 1 & 800 \\ 0 & 1 & 400 \end{array} \right]$$

$$R_1 - R_2 \rightarrow R_1 \left[\begin{array}{cc|c} 1 & 0 & 400 \\ 0 & 1 & 400 \end{array} \right]$$

\therefore order 400 desks from each plant. 

(2) Section 6.5 Problems

#2 Augmented matrix followed by reduction:

2	1	10	15	-5
1	-5	2	15	-10
1	1	6	12	9

(6)

$$R_1 \leftrightarrow R_2 \left[\begin{array}{ccccc|c} 1 & -5 & 2 & 15 & -10 \\ 2 & 1 & 10 & 15 & -5 \\ 1 & 1 & 6 & 12 & 9 \end{array} \right]$$

$$R_2 - 2R_1 \rightarrow R_2 \left[\begin{array}{ccccc|c} 1 & -5 & 2 & 15 & -10 \\ 0 & 11 & 6 & -15 & 15 \\ 0 & 6 & 4 & -3 & 19 \end{array} \right]$$

$$\left(\frac{1}{11}\right)R_2 \rightarrow R_2 \left[\begin{array}{ccccc|c} 1 & -5 & 2 & 15 & -10 \\ 0 & 1 & \frac{2}{11} & -\frac{15}{11} & \frac{15}{11} \\ 0 & 6 & 4 & -3 & 19 \end{array} \right]$$

$$R_3 - 6R_2 \rightarrow R_3 \left[\begin{array}{ccccc|c} 1 & -5 & 2 & 15 & -10 \\ 0 & 1 & \frac{2}{11} & -\frac{15}{11} & \frac{15}{11} \\ 0 & 0 & \frac{8}{11} & \frac{57}{11} & \frac{119}{11} \end{array} \right]$$

$$\left(\frac{11}{8}\right)R_3 \rightarrow R_3 \left[\begin{array}{ccccc|c} 1 & -5 & 2 & 15 & -10 \\ 0 & 1 & \frac{2}{11} & -\frac{15}{11} & \frac{15}{11} \\ 0 & 0 & 1 & \frac{57}{8} & \frac{119}{8} \end{array} \right]$$

$$R_1 - 2R_3 \rightarrow R_1 \left[\begin{array}{ccccc|c} 1 & -5 & 0 & \frac{3}{8} & -\frac{159}{8} \\ 0 & 1 & 0 & -\frac{21}{4} & -\frac{27}{4} \\ 0 & 0 & 1 & \frac{57}{8} & \frac{119}{8} \end{array} \right]$$

$$R_2 - \frac{6}{11}R_3 \rightarrow R_2 \left[\begin{array}{ccccc|c} 1 & -5 & 0 & \frac{3}{8} & -\frac{159}{8} \\ 0 & 1 & 0 & -\frac{21}{4} & -\frac{27}{4} \\ 0 & 0 & 1 & \frac{57}{8} & \frac{119}{8} \end{array} \right]$$

$$R_1 + 5R_2 \rightarrow R_1 \left[\begin{array}{ccccc|c} 1 & 0 & 0 & -\frac{21}{2} & -\frac{147}{2} \\ 0 & 1 & 0 & -\frac{21}{4} & -\frac{27}{4} \\ 0 & 0 & 1 & \frac{57}{8} & \frac{119}{8} \end{array} \right]$$

Basic Variables

Solve for w, x, y in terms of $z (= r)$ (Reduced Matrix) (w, x, y are in columns with leading 1's)

$$w = \frac{51}{2}r - \frac{147}{2}$$

$$x = \frac{21}{4}r - \frac{27}{2}$$

$$y = -\frac{57}{8}r + \frac{119}{8}$$

$$z = r \text{ where } r \in \mathbb{R}.$$

This is the solution to the given system of eqns.



(7)

#6 Write-out augmented matrix, then reduce:

$$\left[\begin{array}{ccccc} 1 & 1 & 1 & 2 & 4 \\ 2 & 1 & 2 & 2 & 7 \\ 1 & 2 & 1 & 4 & 5 \\ 3 & -2 & 3 & -4 & 7 \\ 4 & -3 & 4 & -6 & 9 \end{array} \right] \quad \begin{array}{l} R_2 - 2R_1 \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \\ R_4 - 3R_1 \rightarrow R_4 \\ R_5 - 4R_1 \rightarrow R_5 \end{array} \quad \left[\begin{array}{ccccc} 1 & 1 & 1 & 2 & 4 \\ 0 & -1 & 0 & -2 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & -5 & 0 & -10 & -5 \\ 0 & -7 & 0 & -14 & -7 \end{array} \right]$$

$$(-1)R_2 \rightarrow R_2 \quad \left[\begin{array}{ccccc} 1 & 1 & 1 & 2 & 4 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & -5 & 0 & -10 & -5 \\ 0 & -7 & 0 & -14 & -7 \end{array} \right] \quad \begin{array}{l} R_1 - R_2 \rightarrow R_1 \\ R_3 - R_2 \rightarrow R_3 \\ R_4 + 5R_2 \rightarrow R_4 \\ R_5 + 7R_2 \rightarrow R_5 \end{array} \quad \left[\begin{array}{ccccc} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Isolate variables in columns containing leading 1's $\boxed{1}$ \therefore basic
They are w and x. \therefore variables

(Reduced Matrix)

\therefore solution to the given system is:

$$\left. \begin{array}{l} w = -r + 3 \\ x = -2s + 1 \\ y = r \\ z = s \end{array} \right\} \quad \begin{array}{l} r \text{ and } s \text{ are parameters} \\ \text{and they can take on} \\ \text{any real values.} \\ (\text{This is a 2-parameter family}) \end{array}$$



#10 We use the Theorem on Homogeneous systems on page 275-6 and, in particular, the Corollary there,

$$\begin{array}{ll} 5w + 7x - 2y - 5z = 0 & \text{Two equations and} \\ 7w - 6x + 9y - 5z = 0 & \text{four variables.} \end{array}$$

$$\underbrace{(\# \text{ of Eq } ns)}_{m=2} < \underbrace{(\# \text{ of Unknowns})}_{n=4}$$

\therefore we have infinitely many solutions.



(8)

#12 Homogenous system is:

$$\begin{aligned} 2x + 3y + 12z &= 0 \\ 3x - 2y + 5z &= 0 \\ 4x + y + 14z &= 0 \end{aligned}$$

Corollary on page 276
does not apply
 \therefore obtain reduced form
of coefficient matrix.

$$\left[\begin{array}{ccc|c} 2 & 3 & 12 \\ 3 & -2 & 5 \\ 4 & 1 & 14 \end{array} \right] \left(\frac{1}{2} R_1 \right) \rightarrow R_1 \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 6 \\ 3 & -2 & 5 \\ 4 & 1 & 14 \end{array} \right]$$

$$R_2 - 3R_1 \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 6 \\ 0 & -\frac{13}{2} & -13 \\ 0 & -5 & -10 \end{array} \right] \left(-\frac{2}{13} \right) R_2 \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 6 \\ 0 & 1 & 2 \\ 0 & -5 & -10 \end{array} \right]$$

$$R_3 + 5R_2 \rightarrow R_3 \left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right] R_1 - \frac{3}{2} R_2 \rightarrow R_1 \left[\begin{array}{ccc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right]$$

(= R ... reduced matrix)

R has $k=2$ non-zero rows
and $n=3$ unknowns

$\because k < n$, the Theorem on page 275 says that
the system above has infinitely many solutions.

#16 We only need to reduce the coefficient matrix
for a homogenous system:

$$\left[\begin{array}{cc|c} 2 & -5 \\ 8 & -20 \end{array} \right] \left(\frac{1}{2} R_1 \right) \rightarrow R_1 \left[\begin{array}{cc|c} 1 & -\frac{5}{2} \\ 8 & -20 \end{array} \right]$$

$$R_2 - 8R_1 \rightarrow R_2 \left[\begin{array}{cc|c} 1 & -\frac{5}{2} \\ 0 & 0 \end{array} \right] \therefore \text{solution to the given system is } x = \frac{5}{2}r, r \in \mathbb{R}, y = r$$

(9)

#18 Homogeneous system is : $\begin{cases} 4x + 7y = 0 \\ 2x + 3y = 0 \end{cases}$ (*)

$$\left[\begin{array}{cc|c} 4 & 7 & 0 \\ 2 & 3 & 0 \end{array} \right] \xrightarrow{\frac{1}{4}R_1} \left[\begin{array}{cc|c} 1 & \frac{7}{4} & 0 \\ 2 & 3 & 0 \end{array} \right] \quad R_2 - 2R_1 \rightarrow R_2 \left[\begin{array}{cc|c} 1 & \frac{7}{4} & 0 \\ 0 & -\frac{1}{2} & 0 \end{array} \right]$$

$$(-2)R_2 \rightarrow R_2 \left[\begin{array}{cc|c} 1 & \frac{7}{4} & 0 \\ 0 & 1 & 0 \end{array} \right] \quad R_1 - \frac{7}{4}R_2 \rightarrow R_1 \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \right]$$

\therefore the only solution to (*) is the trivial one,
namely $x=0$ and $y=0$. 

#20 Homogeneous system is : $\begin{cases} 2x + y + z = 0 \\ x - y + 2z = 0 \\ x + y + z = 0 \end{cases}$

Reducing the coefficient

matrix gives the following :

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 1 & 0 \end{array} \right] R_1 \leftrightarrow R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 0 \\ 2 & 1 & 1 & 0 \end{array} \right] R_2 - R_1 \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ 2 & 1 & 1 & 0 \end{array} \right] R_3 - 2R_1 \rightarrow R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array} \right]$$

$$R_2 \leftrightarrow R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -2 & 1 & 0 \end{array} \right] (-1)R_2 \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 \end{array} \right] R_1 - R_2 \rightarrow R_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 \end{array} \right] R_3 + 2R_2 \rightarrow R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{array} \right]$$

$$\left(\frac{1}{3}\right)R_3 \rightarrow R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] R_2 - R_3 \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

\therefore the only solution is the trivial one :

$$x = y = z = 0.$$



(10)

$$\textcircled{3} \quad A = \begin{bmatrix} 1 & -4 & 7 & a \\ 0 & 3 & -5 & b \\ -2 & 5 & -9 & c \end{bmatrix} \quad \begin{array}{l} \text{We row reduce } A \text{ and} \\ \text{expect to get a row of 0's,} \end{array}$$

$R_3 + 2R_1 \rightarrow R_3$

$$\begin{bmatrix} 1 & -4 & 7 & a \\ 0 & 3 & -5 & b \\ 0 & -3 & 5 & \underline{(c+2a)} \end{bmatrix} \quad \begin{array}{l} \text{except for} \\ \text{a non-zero} \\ \text{entry} \end{array}$$

$R_3 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & -4 & 7 & a \\ 0 & 3 & -5 & b \\ 0 & 0 & 0 & \underline{(c+2a+b)} \end{bmatrix}$$

$(\frac{1}{3})R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & -4 & 7 & a \\ 0 & 1 & -\frac{5}{3} & \frac{b}{3} \\ 0 & 0 & 0 & \underline{(c+2a+b)} \end{bmatrix}$$

$R_1 + 4R_2 \rightarrow R_1$

$$\begin{bmatrix} 1 & 0 & \frac{1}{3} & a + \frac{4}{3}b \\ 0 & 1 & -\frac{5}{3} & \frac{b}{3} \\ 0 & 0 & 0 & \underline{(c+2a+b)} \end{bmatrix} = R$$

∴ in order for the linear system having A as its augmented matrix to have infinitely many solutions, we require $2a + b + c = 0$
 (as seen from the last row of R) ■

\textcircled{4} Given system is :
$$\begin{aligned} x + 3y &= a \\ 4x + by &= 8 \end{aligned} \quad \left. \right\} (*)$$

a, b are real constants. Consider the augmented matrix for $(*)$:

$$A = \left[\begin{array}{cc|c} 1 & 3 & a \\ 4 & b & 8 \end{array} \right] \xrightarrow{R_2 - 4R_1} \left[\begin{array}{cc|c} 1 & 3 & a \\ 0 & b-12 & 8-4a \end{array} \right] \quad (11)$$

The second row corresponds to the equation

$$(b-12)y = 8-4a \quad (***)$$

We consider several cases:

(i) No solution: If $b=12$ and $a \neq 2$, then

(***) gives $0y \neq 0$, which has no solution.

(ii) A unique solution: If $b \neq 12$ then

$$(***) \text{ gives } y = \frac{8-4a}{b-12}$$

and the 1st row of A gives $x = a - 3y$

Thus, we do get a unique solution when $b \neq 12$.

(iii) Infinitely many solutions: If $b=12$ and $a=2$,

then (***) gives $0y=0$ which is true for all $y \in \mathbb{R}$. Then the 1st row of A gives $x = a - 3y$

so every choice of y gives a value for x ,
so we have infinitely many solutions. ■

$$(5) \quad A = \left[\begin{array}{ccc} 2 & -1 & 3 \\ 1 & 4 & -6 \\ 2 & 0 & 5 \end{array} \right], \quad B = \left[\begin{array}{cc} 1 & 2 \\ 3 & -7 \\ 1 & 2 \end{array} \right] \quad \text{and} \quad C = AB$$

$$\therefore C = \left[\begin{array}{cc} 2 & 17 \\ 7 & -38 \\ 7 & 14 \end{array} \right] \quad \text{The two columns are} \quad \left[\begin{array}{c} 2 \\ 7 \\ 7 \end{array} \right] + \left[\begin{array}{c} 17 \\ -38 \\ 14 \end{array} \right]$$

We have

$$\begin{bmatrix} 2 \\ 7 \\ 7 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ -6 \\ 5 \end{bmatrix} \quad (12)$$

and

$$\begin{bmatrix} 17 \\ -38 \\ 14 \end{bmatrix} = 2 \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} - 7 \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ -6 \\ 5 \end{bmatrix}$$

sums/differences of scalar multiples of the columns of A

(i.e. Columns of $C = AB$ are written as linear combinations of columns of A with scalars from B)

⑥ $Q = \begin{bmatrix} 200 & 300 & 100 & 200 \\ 100 & 200 & 400 & 0 \end{bmatrix} \begin{matrix} I_1 \\ I_2 \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{matrix}$

$\therefore q_{i,j} = \# \text{ of shares of stock } S_j \text{ held by investor } I_i$
 $j = 1, 2, 3, 4; i = 1, 2$

(a) $V(n) = \begin{bmatrix} v_1(n) \\ v_2(n) \\ v_3(n) \\ v_4(n) \end{bmatrix} = \begin{bmatrix} v_1(n) \\ v_2(n) \\ v_3(n) \\ v_4(n) \end{bmatrix}$

$$\therefore QV(n) = \begin{bmatrix} 200v_1(n) + 300v_2(n) + 100v_3(n) + 200v_4(n) \\ 100v_1(n) + 200v_2(n) + 400v_3(n) \end{bmatrix}$$

1st entry = value of I_1 's portfolio on the n th day of 2011.

2nd entry = value of I_2 's portfolio on the n th day of 2011.

(13)

$$(b) A^T = [1 \ 1 \ 1 \ 1] \text{ so } A = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$QA = \begin{bmatrix} 200 + 300 + 100 + 200 \\ 100 + 200 + 400 \end{bmatrix} = \begin{bmatrix} 800 \\ 700 \end{bmatrix}$$

1st entry = total number of shares of S_1, \dots, S_4
 (combined) held by I_1

2nd entry = total number of shares of S_1, \dots, S_4
 (combined) held by I_2

$$B = [1 \ 1]$$

$$BQ = [300 \ 500 \ 500 \ 200]$$

j^{th} entry = total number of shares of stock
 S_j held by I_1 and I_2 .
 $j = 1, 2, 3, 4$ (combined) 

(∴ $A + B$ are matrices that "add")

— **BIG END** —