

# MATA33 ASSIGNMENT #2 SOLUTIONS

## ① Problems from § 6.1

#2 A is a  $4 \times 4$  matrix so has order 4. (See p. 244)

#4  $A_{42} = 0$  ( $4^{\text{th}}$  row,  $2^{\text{nd}}$  column of A) "order" is used only for square matrices

#6  $A_{34} = 0$  ( $3^{\text{rd}}$  row,  $4^{\text{th}}$  column of A)

#8  $A_{55}$  is not defined (A has only 4 rows and 4 columns, so  $A_{ij}$  is defined only when  $1 \leq i, j \leq 4$ )

#10 We want upper triangular  $U = [u_{ij}]$  where

$u_{ij} = i+j$ ,  $1 \leq i, j \leq 4$ . The upper triangular part requires  $u_{ij} = 0$  for all  $i > j$ . (See page 245)

We now have  $U = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 4 & 5 & 6 \\ 0 & 0 & 6 & 7 \\ 0 & 0 & 0 & 8 \end{bmatrix}$

text uses  $B = [B_{ij}]$

#12<sup>(a)</sup> We construct  $2 \times 2$   $B = [b_{ij}]$  where  $b_{ij} = (-1)^{i-j}(i^2 - j^2)$

$$\therefore B = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} \quad \text{You check that the entries}$$

in B do satisfy  $b_{ij} = (-1)^{i-j}(i^2 - j^2)$ ,  $1 \leq i, j \leq 2$ .

(b) See next page; lower right corner.

#14 (a) diag = 2, 5, -3, 1 } Diagonal entries

(b) diag =  $x^2, \sqrt{y}, 1$  } are of the form  $A_{ii}$

(2)

#18  $A = [2 \ 4 \ 6 \ 8] \Rightarrow A^T = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}$   
 $(A \text{ is } 1 \times 4, A^T \text{ is } 4 \times 1)$

#20  $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 5 & 1 \\ 0 & 1 & 3 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 5 & 1 \\ 0 & 1 & 3 \end{bmatrix}$

$(A \text{ is } 3 \times 3 \Rightarrow A^T \text{ is } 3 \times 3 \text{ too.})$

(We say that  $A$  above is symmetric since  $A = A^T$ ) (See #22 below)

#22 The matrix  $A = \begin{bmatrix} 2 & 5 & -3 & 0 \\ 0 & 3 & 6 & 2 \\ 7 & 8 & -2 & 1 \end{bmatrix}$  is not symmetric because  $A^T \neq A$  (the sizes are inappropriate to be symmetric — symmetric matrices are square! Of course, not all square matrices are symmetric)

#24 Solve  $\begin{bmatrix} 3x & 2y-1 \\ z & 5w \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 7 & 15 \end{bmatrix}$

Using equality of matrices we get some equations:

$$\begin{aligned} 3x &= 9 \rightarrow x = 3 \\ 2y-1 &= 6 \rightarrow y = \frac{7}{2} \\ z &= 7 \rightarrow z = 7 \\ 5w &= 15 \rightarrow w = 3 \end{aligned}$$

#12(b) D is  $2 \times 3$  and

$$D = [d_{ij}] = [(t^1)^i (j^3)]$$

$$= \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -8 & -27 \\ 1 & 8 & 27 \end{bmatrix}.$$



(3)

#26 There is no solution! Equating entries in the 3<sup>rd</sup> row & column gives 7=8, which is false.

#28 Row vector is  $[125 \quad 275 \quad 400]$

Column vector is  $\begin{bmatrix} 0.95 \\ 1.03 \\ 1.25 \end{bmatrix}$



(2) Problems from §6.2

#2 With the matrices given, we have

$$\begin{bmatrix} 2+7+2 & -7-4+7 \\ -6-2+7 & 4+1+2 \end{bmatrix} = \begin{bmatrix} 11 & -4 \\ -1 & 7 \end{bmatrix}$$

$$\#4 \frac{1}{2} \begin{bmatrix} 4 & -2 & 6 \\ 2 & 10 & -12 \\ 0 & 0 & 7 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 5 & -6 \\ 0 & 0 & \frac{7}{2} \end{bmatrix}$$

#6  $[7 \quad 7] + 66$  is not defined. We can add to  $[7 \quad 7]$  only another matrix of size  $1 \times 2$  (or, to the number 66, another number)

$$\begin{aligned} \#8 \begin{bmatrix} 5 & 3 \\ -2 & 6 \end{bmatrix} + 7 \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} &= \begin{bmatrix} 5 & 3 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 3 \\ -2 & 6 \end{bmatrix} \end{aligned}$$

$$\begin{array}{l} \#10 \\ \left[ \begin{array}{cc} 1 & -1 \\ 2 & 0 \\ 3 & -6 \\ 4 & 9 \end{array} \right] - 3 \left[ \begin{array}{cc} -6 & 9 \\ 2 & 6 \\ 1 & -2 \\ 4 & 5 \end{array} \right] = \left[ \begin{array}{cc} 1 & -1 \\ 2 & 0 \\ 3 & -6 \\ 4 & 9 \end{array} \right] - \left[ \begin{array}{cc} -18 & 27 \\ 6 & 18 \\ 3 & -6 \\ 12 & 15 \end{array} \right] \end{array} \quad (4)$$

$$= \left[ \begin{array}{cc} 19 & -28 \\ -4 & -18 \\ 0 & 0 \\ -8 & -6 \end{array} \right]$$

$$\#14 \quad -(A-B) = - \left[ \begin{array}{cc} 8 & 6 \\ 1 & 0 \end{array} \right] = \left[ \begin{array}{cc} -8 & -6 \\ -1 & 0 \end{array} \right]$$

$$\#16 \quad A - B + C = \left[ \begin{array}{cc} 8 & 6 \\ 1 & 0 \end{array} \right] + \left[ \begin{array}{cc} -2 & -1 \\ -3 & 3 \end{array} \right] = \left[ \begin{array}{cc} 6 & 5 \\ -2 & 3 \end{array} \right]$$

$$\#18 \quad \underset{\substack{\text{O} \in \mathbb{R}}}{O(A+B)} = O \left[ \begin{array}{cc} -4 & -4 \\ 5 & -6 \end{array} \right] = \left[ \begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array} \right] = O \quad (\text{O-matrix})$$

$$\#20 \quad A + (C+B) = \left[ \begin{array}{cc} 2 & 1 \\ 3 & -3 \end{array} \right] + \left[ \begin{array}{cc} -8 & -6 \\ -1 & 0 \end{array} \right] = \left[ \begin{array}{cc} -6 & -5 \\ 2 & -3 \end{array} \right]$$

$$\#30 \quad (B-C)^T = \left[ \begin{array}{cc} 0 & 3 \\ 3 & -3 \end{array} \right]^T = \left[ \begin{array}{cc} 0 & 3 \\ 3 & -3 \end{array} \right]$$

$$\#32 \quad 2B + B^T = \left[ \begin{array}{cc} 2 & 6 \\ 8 & -2 \end{array} \right] + \left[ \begin{array}{cc} 1 & 4 \\ 3 & -1 \end{array} \right] = \left[ \begin{array}{cc} 3 & 10 \\ 11 & -3 \end{array} \right]$$

#36 Given the system of linear equations

$$\begin{array}{l} 2x - 4y = 16 \\ 5x + 7y = -3 \end{array}, \text{ the matrix equation is}$$

$$x \left[ \begin{array}{c} 2 \\ 5 \end{array} \right] + y \left[ \begin{array}{c} -4 \\ 7 \end{array} \right] = \left[ \begin{array}{c} 16 \\ -3 \end{array} \right] \quad (\text{A linear combination of columns})$$

(This is not the only "matrix equation" representation)

(5)

#38  $5 \begin{bmatrix} x \\ 3 \end{bmatrix} - 6 \begin{bmatrix} 2 \\ -2y \end{bmatrix} = \begin{bmatrix} -4x \\ 3y \end{bmatrix}$  is given.

$$\begin{bmatrix} 5x - 12 \\ 15 + 12y \end{bmatrix} = \begin{bmatrix} -4x \\ 3y \end{bmatrix}$$

$$\therefore 5x - 12 = -4x \rightarrow 9x = 12 \therefore x = \frac{4}{3}$$

$$15 + 12y = 3y \rightarrow 9y = -15 \therefore y = -\frac{5}{3}$$

$\therefore$  The solution to the given matrix equation  
is  $x = \frac{4}{3}$  and  $y = -\frac{5}{3}$

#42 A represents the 2007 sales (in \$1,000's)  
B " " 2009 " (" ")

$\therefore$  2010 sales are represented by  $2B$

The question should ask, "... what is the  
change in sales between 2007 and 2010  
(not 2003).

Answer is  $2B - A = \begin{bmatrix} 360 & 310 & 290 \\ 470 & 360 & 650 \end{bmatrix}$  

### ③ Problems from § 6.3

#2  $C_{21} = (-2)(0) + (1)(-2) + (-1)(3) = -5$

You can also  
write using

$\sum$  notation:

$$C_{21} = \sum_{k=1}^3 A_{2k} B_{k1}$$

$$= A_{21} B_{11} + A_{22} B_{21} + A_{23} B_{31}$$

#4  $AB = C$      $C = [c_{ij}]$ ,  $A = [a_{ij}]$ ,  $B = [b_{ij}]$     (6)

$$c_{33} = \sum_{k=1}^3 a_{3k} b_{k3} = 0(3) + 4(-2) + 3(-1) = -11$$

#6  $c_{12} = \sum_{k=1}^3 a_{1k} b_{k2} = 1(-2) + 3(4) + (-2)(1) = 8$

#8 D is  $\underbrace{4 \times 3}_{\text{Same}}$  + E is  $\underbrace{3 \times 2}_{\text{Same}}$

$\therefore DE$  is  $\underbrace{4 \times 2}_{\text{Same}}$  so has 8 entries.

#10 D is  $\underbrace{4 \times 3}_{\text{Same}}$  + B is  $\underbrace{3 \times 1}_{\text{Same}}$

$\therefore DB$  is  $4 \times 1$  so has 4 entries.

#20  $\begin{bmatrix} -1 & 1 \\ 0 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 12 & 16 \\ 5 & 0 \end{bmatrix}$

#22  $[2 \ 5 \ 0 \ 1] \begin{bmatrix} 0 \\ 1 \\ 0 \\ -2 \end{bmatrix} = [0 + 5 + 0 - 2] = [3]$

(Note: our final answer here is technically a  $1 \times 1$  matrix:  $[3]$  (and not just the number)  
 $(1 \times 4) (4 \times 1) \rightarrow 1 \times 1$  matrix, not just a number.)

Compare with p. 255 Example 3 (a) and  
 p. 256 Example 5 )

#26  $[1 \ -4] \begin{bmatrix} -2 & 1 \\ 0 & 1 \\ 5 & 0 \end{bmatrix}$  is not defined: 1<sup>st</sup> matrix has size  $1 \times 2$ ; 2<sup>nd</sup> is  $3 \times 2$   
 unequal

(7)

$$\#34 \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}$$

$$\#38 \quad D^2 = DD = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\#46 \quad A^T A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Note that  
 $A^T A$  is  
a symmetric  
matrix.

$$\begin{aligned} \#48 \quad A(B^T)^2 C &= A \left( \begin{bmatrix} 0 & 0 & -1 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}^T \right)^2 C \\ &= A \begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 2 \end{bmatrix}^2 C = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{pmatrix} \begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 2 \end{bmatrix} \end{pmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -3 & 0 \\ -2 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -6 & 3 \\ -4 & 5 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \#58 \quad B^2 - 3B + 2I &= \begin{bmatrix} 0 & 0 & -1 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} - 3B + 2I \\ &= \begin{bmatrix} 0 & 0 & -2 \\ -2 & 1 & -2 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 0 & -3 \\ 6 & -3 & 0 \\ 0 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 & -1 \\ -8 & 6 & -2 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

(8)

#60 We represent a system of linear eq<sup>ns</sup> in matrix form as  $A X = B$  (See p. 261-62)

Here,  $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & -1 & 1 \\ 5 & -1 & 2 \end{bmatrix}$ ,  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 \\ 4 \\ 12 \end{bmatrix}$

so  $A X = B$  is  $\begin{bmatrix} 3 & 1 & 1 \\ 1 & -1 & 1 \\ 5 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 12 \end{bmatrix}$

#64 Let  $N$  represent the number of stocks bought and  $V$  represent the price per share.  $N = [200 \ 300 \ 500 \ 250]$

$$V = \begin{bmatrix} 100 \\ 150 \\ 200 \\ 300 \end{bmatrix}$$

Total cost of all shares is given by  $NV = [200 \ 300 \ 500 \ 250] \begin{bmatrix} 100 \\ 150 \\ 200 \\ 300 \end{bmatrix} = [240,000]$

$\therefore$  total cost is \$240,000

#66 - See the following page.

We say that  $A + B$  "commute"

#68 Let  $A, B$  be matrices for which  $AB$  and  $BA$  are defined and equal :  $AB = BA$   $\left(\because A, B \text{ must be square of same order}\right)$

$$\begin{aligned} \therefore (A+B)(A-B) &= A^2 - AB + BA - B^2 \\ &= A^2 - AB + AB - B^2 = A^2 - B^2 \end{aligned}$$

8.5

#66 Refer to Example 9 on pages 257 – 258.

We have  $R = \begin{bmatrix} 5 & 20 & 16 & 7 & 17 \\ 7 & 18 & 12 & 9 & 21 \\ 6 & 25 & 8 & 5 & 13 \end{bmatrix}$  from the

top of page 258 and C as in the question.

$$(a) RC = \begin{bmatrix} 124,750 & 2,920 \\ 139,250 & 2,540 \\ 113,250 & 2,400 \end{bmatrix}$$

↑                                   ↑

Purchase price of houses      Transportation cost of houses.

Rows give purchase price and transportation cost for each of the 3 house types.

$$(b) QRC = Q(RC) = [5 \ 7 \ 12] (RC)$$

as found in part (a)

$$\begin{aligned} &\uparrow \\ &(Q = [5 \ 7 \ 12] \text{ bottom page 257}) \\ &= [2,957,500 \quad 61,180] \\ &\quad \underbrace{\hspace{1cm}}_{\text{total purchase price for all 3 houses}} \quad \underbrace{\hspace{1cm}}_{\text{total transportation costs.}} \end{aligned}$$

$$(c) Z = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

as in part (b)

$$\begin{aligned} QRCZ &= (QRC) \begin{bmatrix} ? \\ ? \end{bmatrix} = [2,957,500 + 61,180] \\ &= [3,018,680] \end{aligned}$$

Total of purchase price (i.e. material costs)

plus transportation costs for all 3 house types in the "Q order" is \$ 3,018,680.

An interesting application of matrices!



(9)

④ We want an example of  $2 \times 2$  matrices  $A$  &  $B$  such that  $A^2 - B^2 \neq (A-B)(A+B)$ .

Solution: From p. 263 # 68, look for  $A$  &  $B$  that "do not commute" (i.e.  $AB \neq BA$ )

How about, for example, the matrices on

page 255 Example 4:  $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -2 & 1 \\ 1 & 4 \end{bmatrix}$

It is shown there that

$$AB = \begin{bmatrix} -5 & -2 \\ -5 & 7 \end{bmatrix} \text{ and } BA = \begin{bmatrix} -1 & 3 \\ 14 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 9 & -2 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} -2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 17 \end{bmatrix}$$

$$A^2 - B^2 = \begin{bmatrix} 1 & -3 \\ 9 & -2 \end{bmatrix} - \begin{bmatrix} 5 & 2 \\ 2 & 17 \end{bmatrix} = \begin{bmatrix} -4 & -5 \\ 7 & -19 \end{bmatrix}$$

$$(A - B)(A + B) = \begin{bmatrix} 4 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -8 & -10 \\ -12 & -15 \end{bmatrix}$$

$$\therefore A^2 - B^2 \neq (A - B)(A + B)$$

(You should invent other examples-not using ones from the text).



(10)

- ⑤ For part(a), we want  $A, B, C$   $2 \times 2$  matrices such that  $AB = AC$  but  $B \neq C$ .

How about :  $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$ ,  $C = \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}$

We have  $A, B, C \neq 0$ ,  $B \neq C$  and

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = AC$ . There are infinitely many examples. You find some others!

For part(b),  $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}$  work.

We have  $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$  yet  $A, B \neq 0$

Again, there are infinitely many examples;  
find your own:



- ⑥  $A = \begin{bmatrix} 2 & 5 \\ 3 & 9 \end{bmatrix}$   $B = \begin{bmatrix} 6 & -55 \\ -33 & 4 \end{bmatrix}$  are given

$$\begin{aligned} (a) (A - 2I)^T &= \left( \begin{bmatrix} 2 & 5 \\ 3 & 9 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right)^T \\ &= \begin{bmatrix} 0 & 5 \\ 3 & 7 \end{bmatrix}^T = \begin{bmatrix} 0 & 3 \\ 5 & 7 \end{bmatrix} \end{aligned}$$

- (b) We want all  $2 \times 2$  diagonal matrices  $D$  so that  $D^2 - A^2 = B$

(11)

Solution: Consider  $D = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}$  and

$$\begin{aligned} D^2 &= A^2 + B = \begin{bmatrix} 2 & 5 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 9 \end{bmatrix} + \begin{bmatrix} 6 & -55 \\ -33 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 19 & 55 \\ 33 & 96 \end{bmatrix} + \begin{bmatrix} 6 & -55 \\ -33 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 25 & 0 \\ 0 & 100 \end{bmatrix} \end{aligned}$$

$D^2 = \begin{bmatrix} d_1^2 & 0 \\ 0 & d_2^2 \end{bmatrix} \therefore$  we want  $d_1^2 = 25$  and  
 $d_2^2 = 100$ ;  $d_1, d_2 \in \mathbb{R}$ .

$$\therefore d_1 = \pm 5, d_2 = \pm 10$$

$\therefore$  we have 4 possibilities for  $D$ :

$$\begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}, \begin{bmatrix} -5 & 0 \\ 0 & 10 \end{bmatrix}, \begin{bmatrix} 5 & 0 \\ 0 & -10 \end{bmatrix}, \begin{bmatrix} -5 & 0 \\ 0 & -10 \end{bmatrix}.$$


⑦ Here are SIX  $2 \times 2$  matrices whose square is  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ :

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, A_3 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, A_4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad A_6 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(Can you find others?)

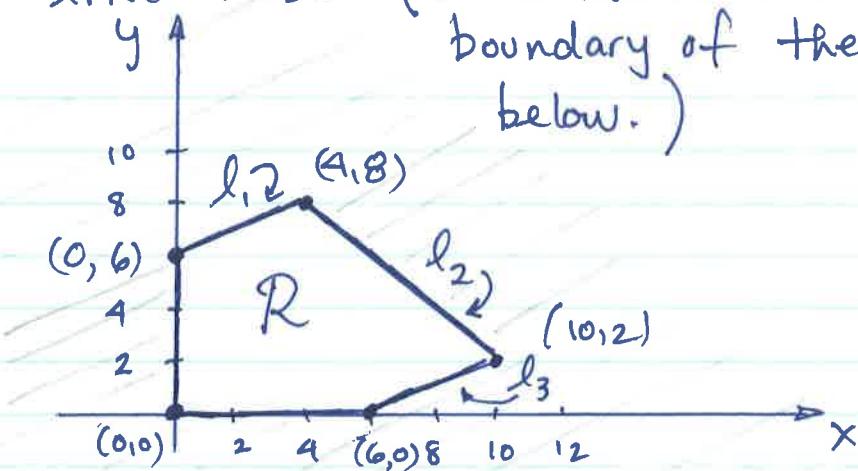


⑧  $R$  has corner points (vertices)  $(0,0), (0,6), (4,8), (10,2)$  and  $(6,0)$ .

*Hilary*

(12)

$R$  looks like this: (it is the interior + boundary of the figure below.)



$$\text{For } l_1 : m = \frac{8-6}{4-0} = \frac{1}{2} \quad \therefore l_1 \text{ is } y = \frac{1}{2}x + 6$$

$$\text{For } l_2 : m = \frac{8-2}{4-10} = -1 \quad \therefore y = -(x-4) + 8 \\ = -x + 12$$

$$\text{For } l_3 : m = \frac{2-0}{10-6} = \frac{1}{2} \quad \therefore y = \frac{1}{2}(x-6) \\ = \frac{1}{2}x - 3$$

Five inequalities whose solution set is  $R$  is the system:

$$-\frac{1}{2}x + y \leq 6 \quad ① \text{ (from } l_1\text{)}$$

$$x + y \leq 12 \quad ② \text{ (from } l_2\text{)}$$

$$-\frac{1}{2}x + y \geq -3 \quad ③ \text{ (from } l_3\text{)}$$

$$x, y \geq 0 \quad ④ \text{ (non-negativity)}$$

As a check, note that  $(4, 4) \in R$  and satisfies every inequality. (You think about how we produce the inequalities from the equations) ■

(13)

⑨ Here is a diagram of  $R$ :

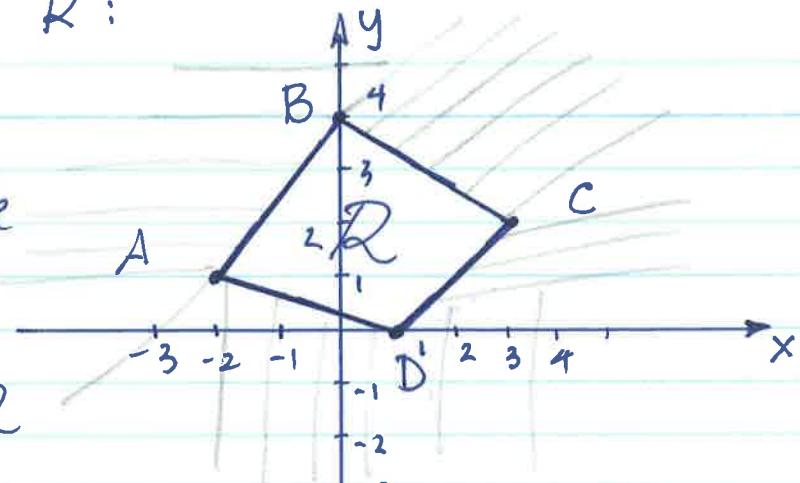
$$A = (-2, 1)$$

$$B = (0, 4)$$

$$C = (3, 2)$$

$$D = (1, 0)$$

Edges,  
Vertices,  
and the  
"interior"  
of  $R$   
are all  
part of  $R$



Let  $r > 0$  be a constant and consider the objective function

$$g(x, y) = Z = rx + 3ry$$

$R$  is  $\neq \emptyset$  and bounded, and  $g$  is linear.

$\therefore$  we optimize  $g$  by corner point evaluation (FTLP)

$$g(A) = -2r + 3r = r \quad g(C) = 9r$$

$$g(B) = 12r \quad g(D) = r$$

$\because r > 0$ ,  $g$  is maximized at  $B$  with value  $12r$

Also, since  $g(A) = g(D) = r$  gives minimal values at two vertices of an edge of  $R$ , it follows from multiple optimal solution theory that  $g$  is minimized at every point on segment  $\overline{AD}$