

① Section 7.1 Problems

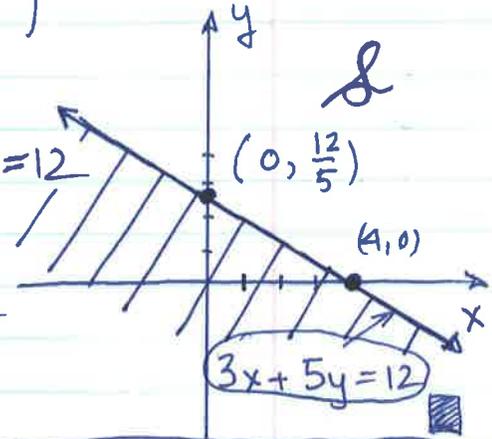
#6 Inequality is $3x + 5y \geq 12$
 Intercepts are $(4, 0)$ & $(0, \frac{12}{5})$

\mathcal{S} is the feasible set.

Solution \mathcal{S} consists of points on or above the line $3x + 5y = 12$

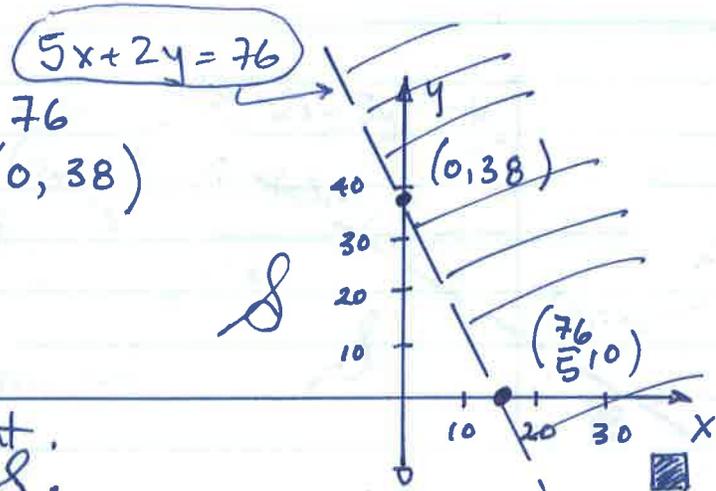
\mathcal{S} is the un-shaded portion (this is consistent with the text convention — see p. 296)

\mathcal{S} is standard but unbounded.



#8 Inequality is $5x + 2y < 76$
 Intercepts are $(\frac{76}{5}, 0)$ & $(0, 38)$

Solution \mathcal{S} consists of points (x, y) below the line. This is seen using $(0, 0)$ as a test point.
 Note that intercepts $\notin \mathcal{S}$.

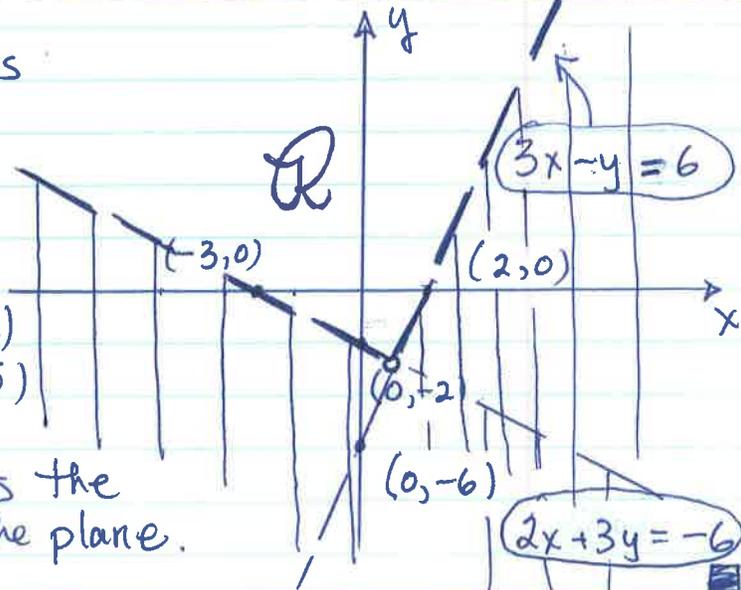


#10 System of inequalities is

- ① $2x + 3y > -6$
 - ② $3x - y < 6$
- (*)

Intercepts: ① $(-3, 0), (0, -2)$
 ② $(2, 0), (0, 6)$

\mathcal{R} is the solution and is the non-shaded part of the plane.



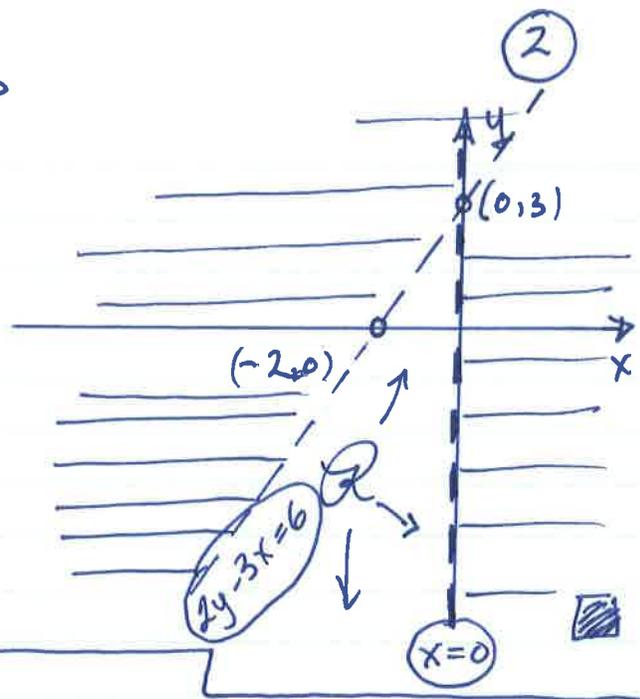
#12 System of inequalities is

$$\begin{cases} \textcircled{1} & 2y - 3x < 6 \\ \textcircled{2} & x < 0 \end{cases} (*)$$

Intercepts for $\textcircled{1}$ are $(-2, 0)$
and $(0, 3)$

\mathcal{R} is the solution to $(*)$

\mathcal{R} is the feasible region.



#18 System of inequalities is

$$(*) \begin{cases} x + 2y \leq 10 & \textcircled{1} \\ 3x + 2y \leq 14 & \textcircled{2} \\ x \geq 1 & \textcircled{3} \\ y \geq 2 & \textcircled{4} \end{cases}$$

Intercepts for $\textcircled{1}$: $(10, 0), (0, 5)$

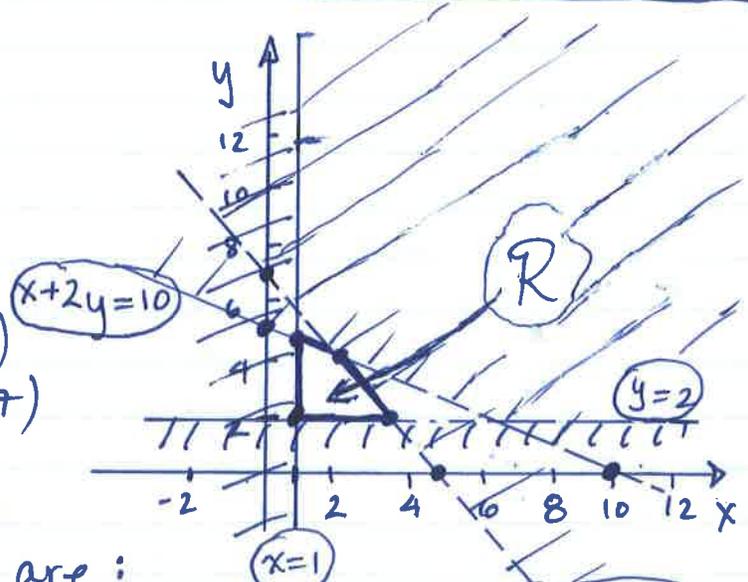
Intercepts for $\textcircled{2}$: $(\frac{14}{3}, 0), (0, 7)$

\mathcal{R} is the solution to $(*)$.

The corner points of \mathcal{R} are:

$(1, 2), (1, \frac{9}{2}), (\frac{10}{3}, 2)$ and $(2, 4)$ (from finding the intersection of lines for $\textcircled{1}$ & $\textcircled{2}$).

Note: corner pts, "edges", and "interior" all $\in \mathcal{R}$.
 $\therefore \mathcal{R}$ is standard.



#22 System of inequalities is

$$(*) \begin{cases} 2x - 3y > -12 & \textcircled{1} \\ 3x + y > -6 & \textcircled{2} \\ y > x & \textcircled{3} \end{cases}$$

Intercepts:

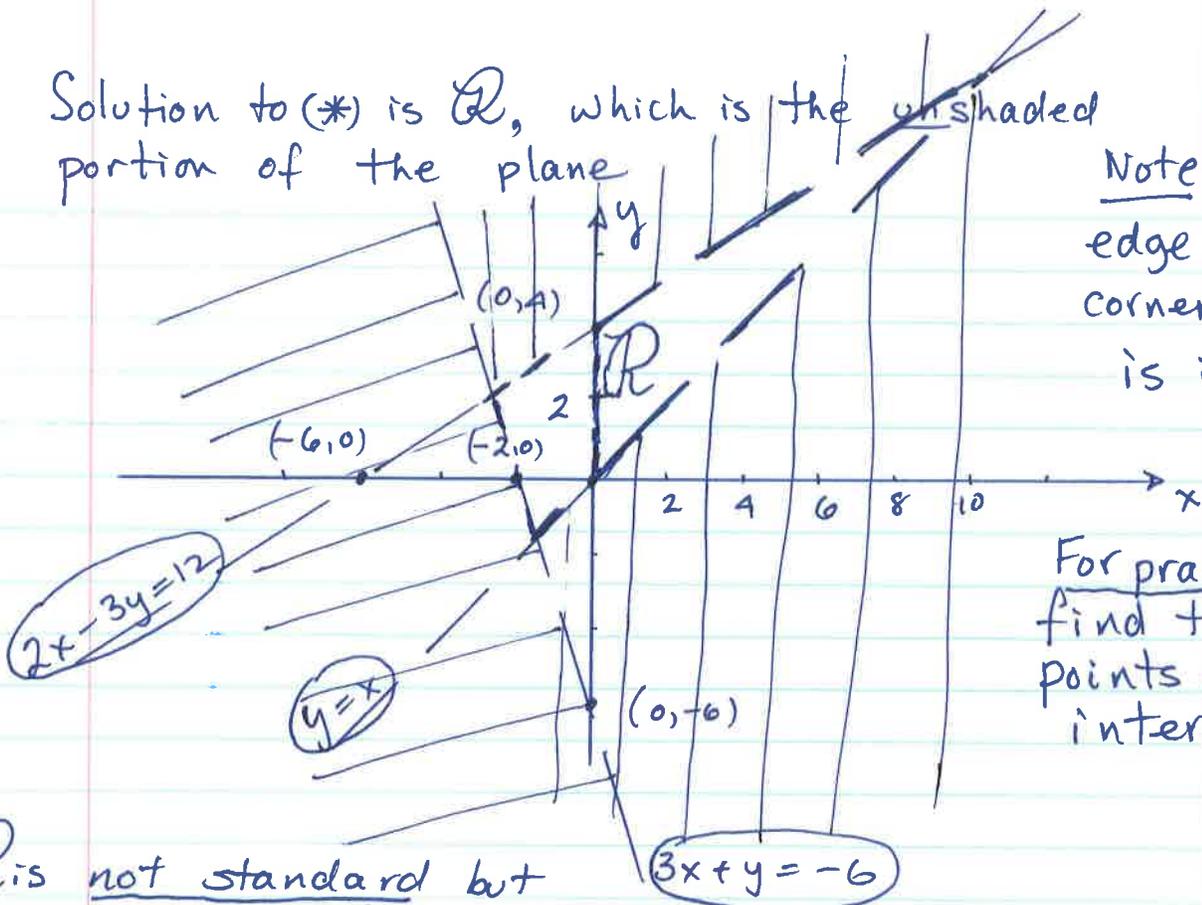
$\textcircled{1}$: $(-6, 0), (0, 4)$

$\textcircled{2}$: $(-2, 0), (0, -6)$

Solution to (*) is \mathcal{Q} , which is the unshaded portion of the plane

(3)

Note: no edge or corner point is in \mathcal{Q} .



For practice: find the points of intersection.

\mathcal{Q} is not standard but is bounded.

#26 Price of products is $p_1 = 7, p_2 = 3$. $x =$ amount of product 1
 Cap on spending is $P = 25$ $y =$ " " " 2

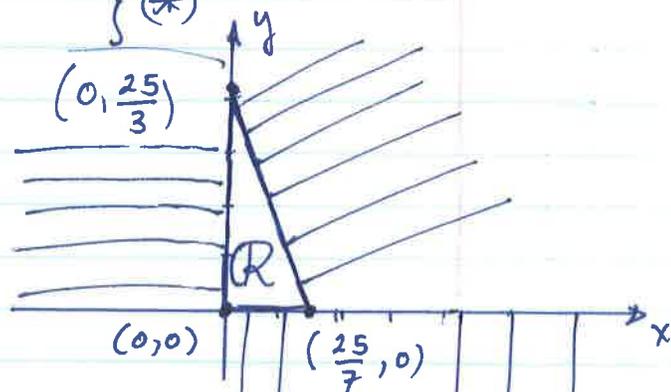
Inequalities determining combinations of purchases:

(*) $7x + 3y \leq 25$

Non-negativity conditions $x, y \geq 0$

Feasible region for (*):

Intercepts $(\frac{25}{7}, 0), (0, \frac{25}{3})$



Region \mathcal{Q} is the interior & boundary of the triangle with vertices $(0, 0), (0, \frac{25}{3})$ and $(\frac{25}{7}, 0)$. \mathcal{Q} is bounded & standard.

#28 System of inequalities is:

$x + y \leq 750, x \geq 0, y \geq 0$

Section 7.2 Problems

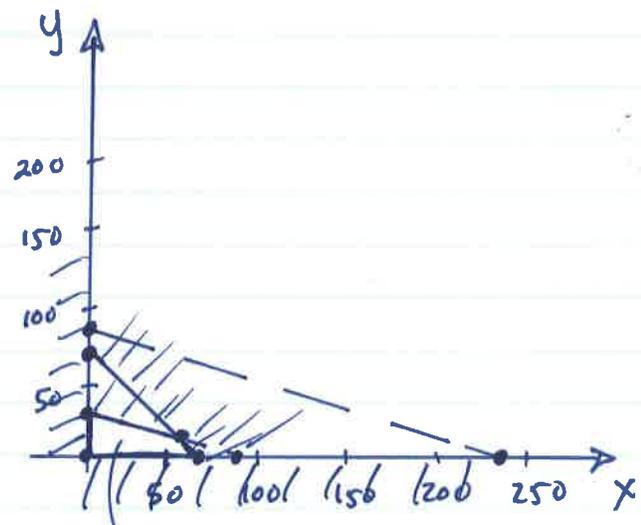
#2 We maximize $P = 3x + 2y$ subject to

$$\begin{aligned}
 x + y &\leq 70 & \textcircled{1} \\
 x + 3y &\leq 240 & \textcircled{2} \\
 x + 3y &\leq 90 & \textcircled{3} \\
 x, y &\geq 0
 \end{aligned}$$

Note how $\textcircled{3}$ implies $\textcircled{2}$

1st get the feasible region \mathcal{R}

- $\textcircled{1}$: Intercepts: $(70, 0), (0, 70)$
- $\textcircled{2}$ $(240, 0), (0, 80)$
- $\textcircled{3}$ $(90, 0), (0, 30)$



Lines for $\textcircled{1}$ & $\textcircled{3}$ cross.

Intersection:

$$\begin{aligned}
 x + y &= 70 \\
 x + 3y &= 90
 \end{aligned}$$

Subtract $2y = 20 \rightarrow y = 10$
 $x = 60$

Corner points of \mathcal{R} are $(0, 0), (0, 30), (60, 10), (70, 0)$

\mathcal{R} = feasible region $\neq \emptyset$ and bounded and standard.

By FTLP (Page 301) we maximize by corner point eval.ⁿ

$$P(0, 0) = 0 \quad P(0, 30) = 60 \quad P(60, 10) = 200 \quad P(70, 0) = 210$$

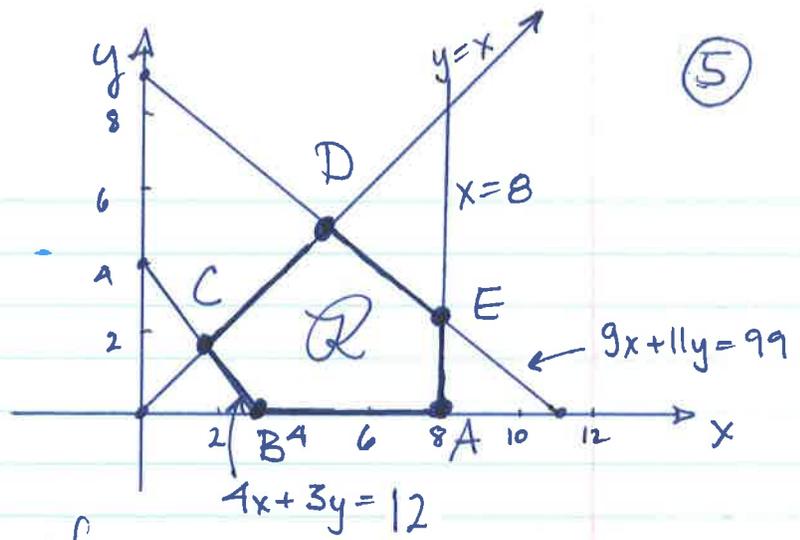
Conclude that max value of P over \mathcal{R} is 210 at the point $(70, 0)$. ▣

#4 We minimize $Z = x + y$ subject to

$$\begin{aligned}
 \textcircled{1} \quad x - y &\geq 0 & x &\leq 8 & \textcircled{4} \\
 \textcircled{2} \quad 4x + 3y &\geq 12 \\
 \textcircled{3} \quad 9x + 11y &\leq 99 & x, y &\geq 0 & \textcircled{5}
 \end{aligned}$$

Intercepts are:

- ① (0,0)
- ② (3,0) + (0,4)
- ③ (11,0) + (0,9)
- ④ (8,0)



For clarity we describe R rather than non-shade

The feasible region is R . It is the interior and boundary of the 5-sided figure with vertices (R is an example of a CONVEX SET)
 A, B, C, D, E where

$A = (8,0), B = (3,0), E = (8, \frac{27}{11})$. Details:

For E let $x=8$ in $9x+11y=99$. We then get
 $11y = 99 - 72 = 27$, so $y = \frac{27}{11}$ as in E .

$C = (\frac{12}{7}, \frac{12}{7})$. To see why note that C is the point of intersection of lines $y=x$ and $4x+3y=12$.
 Let $y=x$ and sub-in. We get $7x=12$ so $x = \frac{12}{7}$ and then $y = \frac{12}{7}$ too. $\therefore C = (\frac{12}{7}, \frac{12}{7})$

Lastly for D sub $y=x$ into $9x+11y=99$
 We get $20x=99$, so $x = \frac{99}{20}$ and $y = \frac{99}{20}$ too.
 $\therefore D = (\frac{99}{20}, \frac{99}{20})$.

R is non-empty + bounded + Z is linear. To minimize, evaluate at the corner pts: (used FTLP) (p.301)

$Z(A) = 8$

$Z(D) = \frac{99}{10} = 9.9$

$Z(B) = 3$

$Z(C) = \frac{24}{7} \sim 3.42$ $Z(E) = \frac{115}{11} \sim 10.45$

\therefore minimum value is 3 at $B = (3,0)$. ▣

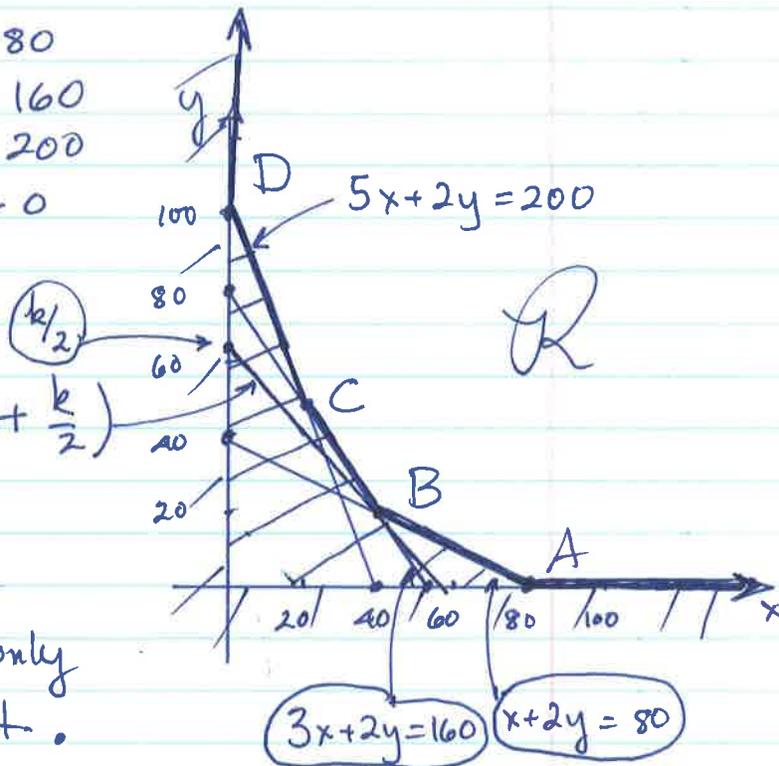
#10 We seek to minimize the objective function $C = 2x + 2y$ subject to the constraints

(7)

- ① $x + 2y \geq 80$
- ② $3x + 2y \geq 160$
- ③ $5x + 2y \geq 200$
- ④ $x, y \geq 0$

Intercepts:

- ① $(80, 0), (0, 40)$
- ② $(\frac{160}{3}, 0), (0, 80)$
- ③ $(40, 0), (0, 100)$
- ④ \Rightarrow once again we need only consider the 1ST quadrant.



Since ① - ③ involve inequalities of the form $ax + by \geq c$ where $a, b, c > 0$ we consider the region for each as on (or above) its defining line.

\therefore the feasible region R is unbounded and has corner points A, B, C, D .

(for clarity I have shaded the non-region)

Coordinates of corner points are

$A = (80, 0)$ $B = (40, 20)$ (Intersection of lines $x + 2y = 80$ & $3x + 2y = 160$)

$C = (20, 50)$ (Intersection of lines $3x + 2y = 160$ & $5x + 2y = 200$)

$D = (0, 100)$

Even though R is unbounded C will (in this case) have a minimum on R .

To prove why $C = 2x + 2y$ has a minimum ⑧
 on \mathcal{R} we argue in a way that is similar
 to the "isoprofit" concept on p.300 & p.304.

Fix $k > 0$ and let $2x + 2y = k$. We re-write
 as $y = -x + \frac{k}{2}$. This line has slope -1
 and intersects \mathcal{R} with the smallest value
of k at the point $B = (40, 20)$.

(see the diagram). We have $y = 20, x = 40$
 so $20 = -40 + \frac{k}{2} \Rightarrow k = 120$.

\therefore the minimum value of C is 120 at the
 corner point $B = (40, 20)$. ▣

③

We seek to minimize the objective function

$Z = y - x$ subject to

- ① $x \geq 3$
- ② $x + 3y \geq 6$
- ③ $x - 3y \geq -6$
- ④ $x, y \geq 0$

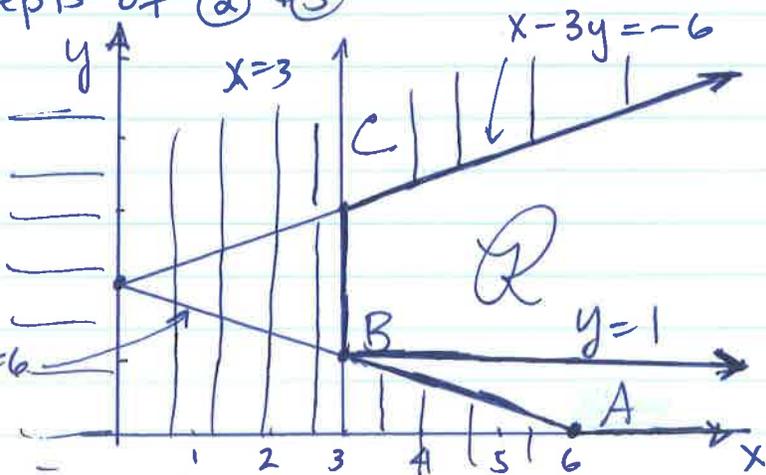
To get the feasible region we need
 only get the intercepts of ② & ③

② $(6, 0), (0, 2)$

③ $(-6, 0), (0, 2)$

From ④, the feasible

region lies in the $x + 3y = 6$
 1st quadrant.



As usual the feasible region R is the unshaded portion of the plane.

R is unbounded. In this case Z has no minimum value. To see why, note that we can choose points in R that make Z arbitrarily negative.

For example, with corner points $A = (6, 0)$, $B = (3, 1)$ and $C = (3, 3)$, we see that the horizontal line $y = 1$ lies in R for all $x \geq 3$ (see the diagram) ($y = 1$ is bold for clarity).
Restrict Z to this line to get $Z = 1 - x$.

We see that as $x \rightarrow +\infty$, $Z \rightarrow -\infty$ so Z has no minimum value on R . 

4 Section 7.2 Problems

#14 View the table of information given in the question.

Let $x = \#$ of Vista made per day $\rightarrow x \geq 0$
 $y = \#$ of Xtreme made per day $\rightarrow y \geq 0$

The profit function is $P = 50x + 80y$

- Constraints are
- $x + 3y \leq 24$ ①
 - $2x + 2y \leq 24$ ②
 - $x, y \geq 0$ ③

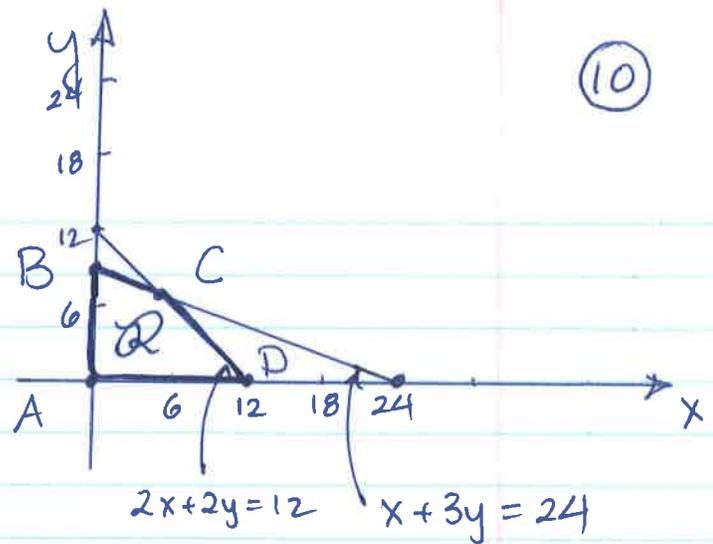
Important } ① is derived from the left column in the table,
* } ② is derived from the right column in the table.

Intercepts for ① and ②

① $(24, 0), (0, 8)$

② $(12, 0), (0, 12)$

$x, y \geq 0$ in ③ means
we only need consider
the 1ST quadrant.



The feasible region R is the 4-sided bounded region with corner points

$A = (0, 0)$ $B = (0, 8)$ $C = (6, 6)$

and $D = (12, 0)$

(Intersection of lines
 $2x + 2y = 12$ & $x + 3y = 24$)

(Since both ① + ② have the form $ax + by \leq c$ where $a, b, c > 0$, the region R is that set of points simultaneously lying "below/on" the lines that underly ① & ②).

standard P is a linear objective function & R is non-empty and bounded, so we maximize P by corner point evaluation: (FTLP again!)

$P(A) = 0$

$P(C) = 780$

$P(B) = 640$

$P(D) = 600$

\therefore Profit is maximized at $C = (6, 6)$ and has value 780. Thus make 6 Vista and 6 Xtreme models.

(A nice "applied problem")



Another interesting applied LP problem. (11)
#18 Carefully read (and re-read) all information in the question. We let

$$\begin{aligned}x &= \# \text{ of days Refinery I operates} & x \geq 0 \\y &= \# \text{ of days Refinery II operates} & y \geq 0\end{aligned}$$

The cost data in the question informs that the cost function (to be minimized) is

$$C = 25,000x + 20,000y.$$

x, y are subject to the constraints (obtained by careful reading):

$$\text{(low)} \quad 2000x + 1000y \geq 8000$$

$$\text{(medium)} \quad 3000x + 2000y \geq 14,000$$

$$\text{(high)} \quad 1000x + 1000y \geq 5000$$

$$x, y \geq 0. \quad (\text{Non negativity conditions})$$

The 1st three constraints above can be simplified by dividing by 1000. We get

$$\text{(low)} \quad 2x + y \geq 8 \quad (1)$$

$$\text{(med)} \quad 3x + 2y \geq 14 \quad (2)$$

$$\text{(high)} \quad 1x + 1y \geq 5 \quad (3)$$

$$x, y \geq 0.$$

Intercepts:

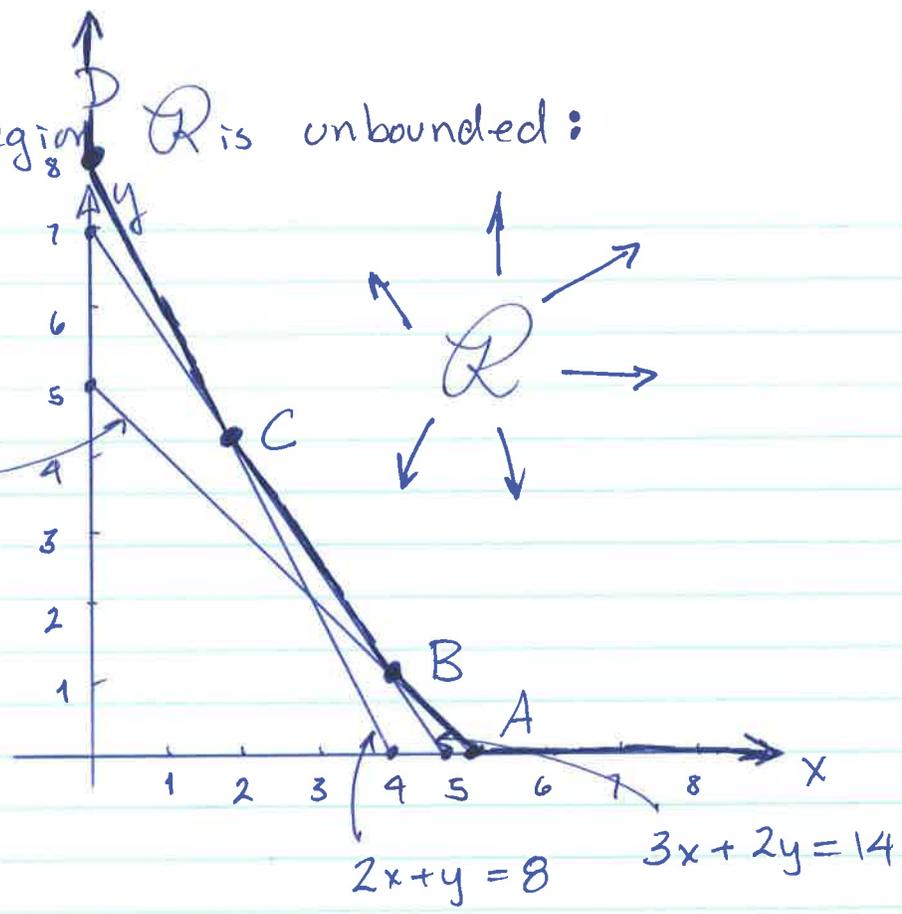
$$(1) \quad (4, 0), (0, 8)$$

$$(2) \quad \left(\frac{14}{3}, 0\right), (0, 7)$$

$$(3) \quad (5, 0), (0, 5)$$

Again since $x, y \geq 0$, our feasible region lies in 1st quadrant.

Feasible region R is unbounded:



No shading - just for clarity.

R lies to top/right of A, B, C, D in 1st quadrant.

$A = (5, 0)$

$B = (4, 1)$ (Intersection of $x + y = 5$ and $3x + 2y = 14$)

$C = (2, 4)$ (Intersection of $2x + y = 8$ and $3x + 2y = 14$)

$D = (8, 0)$

To see that C is actually minimized on (unbounded) R , use the "isoprofit" ideas again on p. 300-304.

For $k \geq 0$ fixed and $C = 25,000x + 20,000y = k$

we get $y = -\frac{5}{4}x + \frac{k}{20,000}$. This line crosses

R with smallest value of k at the point B .

We double check this by evaluation :

$$C(A) = 125,000 \quad C(C) = 130,000$$

$$C(B) = 120,000 \quad C(D) = 160,000$$

∴ Cost is minimized at \$120,000 when Refinery I operates for 4 days and Refinery II for 1 day. ▣

⑤ Section 7.3 Problems

#2 We seek to maximize $Z = 2x + 2y$ subject to the constraints

- ① $2x - y \geq -4$
- ② $x - 2y \leq 4$
- ③ $x + y = 6$
- ④ $x, y \geq 0$

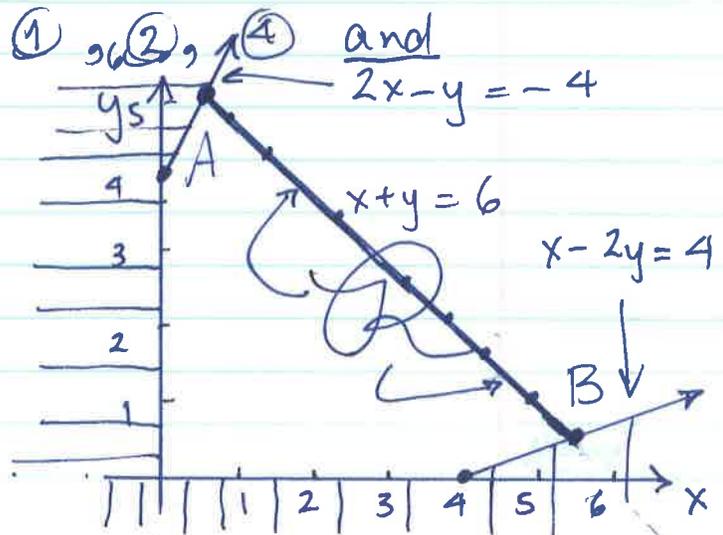
Note: ③ is a line, not an inequality.

The feasible region consists of points (x, y) satisfying inequalities ① and ②, and equation ③.

Intercepts for ① and ②

- ① $(0, 4), (-2, 0)$
- ② $(4, 0), (0, -2)$

By ④, we only consider 1st quadrant.



The feasible region \mathcal{R} is ONLY SEGMENT \overline{AB} : (14)

$$A = \left(\frac{2}{3}, \frac{16}{3} \right) \quad \left(\begin{array}{l} \text{Intersection of lines} \\ x+y=6 \text{ and } 2x-y=-4 \end{array} \right)$$

$$B = \left(\frac{16}{3}, \frac{2}{3} \right) \quad \left(\begin{array}{l} \text{Intersection of lines} \\ x+y=6 \text{ and } x-2y=4 \end{array} \right)$$

(ie The set of points satisfying (1) - (4) is only the line segment \overline{AB} .)

$$Z(A) = 12$$

$$Z(B) = 12$$

$\therefore Z$ is maximized at every point on the line segment \overline{AB} . 

(Note that \mathcal{R} is bounded & standard)

#4 (An interesting and difficult problem)

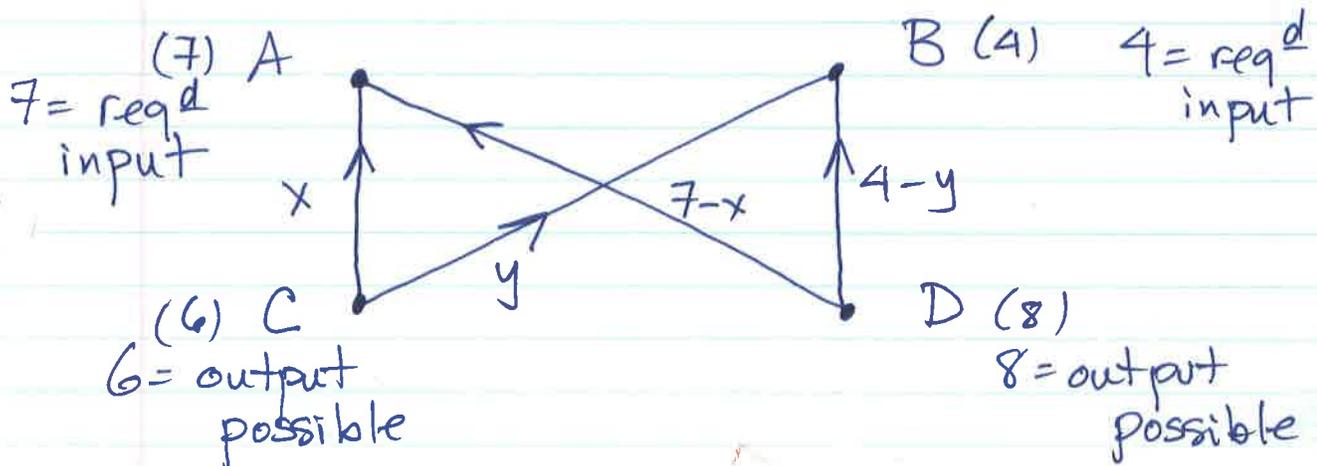
Read (and re-read) the question. A diagram helps to organize all of the given information.

A = Atherton

B = Berkeley

C = Concord

D = Dublin



$x = \#$ of cars delivered from C to A

(15)

$y = \#$ of cars " " C to B

$\therefore 7 - x = \#$ of cars delivered from D to A

$4 - y = \#$ of " " " D to B

(See the diagram)

$Z = \text{Cost function} = 60x + 45y + 50(7-x) + 35(4-y)$

Note that Z is not strictly linear (b/c of 490) } $Z = 350 + 140 + 10x + 10y$
} $Z = 490 + 10x + 10y$.

We seek to minimize Z (cost) subject to the constraints:

① $x + y \leq 6$ (Delivery out of C)

② $x + y \geq 3$ (Delivery out of D)

③ $0 \leq x \leq 7$ ($x \geq 0$ and $7 - x \geq 0$)

④ $0 \leq y \leq 4$ ($y \geq 0$ and $4 - y \geq 0$)

Feasible region is \mathcal{R}
(interior + boundary)

$A = (0, 3)$

$B = (0, 4)$

$C = (2, 4)$

$D = (6, 0)$

$E = (3, 0)$ Evaluate Z at each:

$Z(A) = 520$

$Z(D) = 550$

$Z(B) = 530$

$Z(E) = 520$

$Z(C) = 550$

$\therefore \text{Cost}(Z)$ is minimized at every point on segment AE .

BIG END

