A monopoly has two products, x and y, with demand functions

$$x = 72 - \frac{1}{2} P_x$$

$$y = 120 - P_y$$

where all prices are in hundreds of dollars. The total cost function is

$$C = C(x,y) = x^2 + x\,y + y^2 + 35$$

and the maximum daily output is 40 units.

Find the maximizing level of output, prices and profit.

Rewriting the demand functions we have

$$P_x = 144 - 2x$$

$$P_y = 120 - y$$

and the profit is $\Pi = x\,P_x + y\,P_y - C$

$$= x(144 - 2x) + y(120 - y) - (x^2 + x\,y + y^2 + 35)$$

$$= 144x - 3x^2 - xy - 2y^2 + 120y - 35\ .$$

We want to maximize Π subject to the constraint $x + y = 40$. To do this we

define $F(x, y, \lambda) = 144x - 3x^2 - xy - 2y^2 + 120y - 35 - \lambda(x + y - 40)$.

Now $F_x = 144 - 6x - y - \lambda = 0$

$F_y = -x - 4y + 120 - \lambda = 0$

$F_\lambda = -(x + y - 40) = 0$

Subtracting the second from the first we have $24 - 5x + 3y = 0 \implies y = \frac{-24 + 5x}{3}$. Substituting into the third we have $x + \frac{24 - 5x}{3} = 40 \implies 3x - 24 + 5x = 120 \implies 8x = 144 \implies x = 18 \implies y = \frac{1}{3}(-24 + 90) = 22$.

Hence $P_x = 144 - 2(18) = 108$, $P_y = 120 - 22 = 98$ and $\Pi = 144(18) - 3(18)^2 - (18)(22) - 2(22)^2 + 120(22) - 35 = 2861$ all in hundreds of dollars.