Let f be a demand function for a product A, $q_A = f(p_A, p_B)$.

q_A depends on the per unit prices of both product A and product B.

Definition:

The **partial elasticity of demand for** A with respect to p_A,

denoted η_{p_A}, is defined as

$$\eta_{p_A} = \left(\frac{p_A}{q_A}\right)\left(\frac{\partial q_A}{\partial p_A}\right)$$

The **partial elasticity of demand for** A with respect to p_B,

denoted η_{p_B}, is defined as

$$\eta_{p_B} = \left(\frac{p_B}{q_A}\right)\left(\frac{\partial q_A}{\partial p_B}\right)$$

Let $z = f(x, y)$ be a function of 2–variables. We note that the partial derivatives $f_x(x, y)$ and $f_y(x, y)$ are also functions of 2–variables, so we can take the partial derivatives of these functions:

$$(f_x)_x\,(x, y) \qquad\qquad (f_x)_y\,(x, y)$$

$$(f_y)_x\,(x, y) \qquad\qquad (f_y)_y\,(x, y)$$

These are called the **second-order partial derivatives of** f.

Notation: We write

$$f_{xx} \text{ for } (f_x)_x$$

$$f_{xy} \text{ for } (f_x)_y$$

$$f_{yx} \text{ for } (f_y)_x$$

$$f_{yy} \text{ for } (f_y)_y$$

Since the second-order partials are also functions of 2–variables, we can again take the partials:

$$(f_{xx})_x\,(x,y) \text{ denoted } f_{xxx}\,(x,y)$$

$$(f_{xx})_y\,(x,y) \text{ denoted } f_{xxy}\,(x,y)$$

$$(f_{xy})_x\,(x,y) \text{ denoted } f_{xyx}\,(x,y)$$

etc

These are called the **third-order partial derivatives of** f.

We can continue the process to get even higher order partial derivatives.

Using the alternate notation:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = f_{xx}$$

$$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = f_{xy}$$

$$\frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = f_{yx}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = f_{yy}$$

Going to the third partials:

$$\frac{\partial^3 f}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x^2}\right) = f_{xxx}$$

$$\frac{\partial^3 f}{\partial y\,\partial x^2} = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial x^2}\right) = f_{xxy}$$

$$\frac{\partial^3 f}{\partial x^2\,\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x\,\partial y}\right) = f_{yxx}$$

etc

Definition: The partial derivatives f_{xy} and f_{yx} are called **mixed partial derivatives**.

- If all 2nd order partials are continuous, the mixed 2nd order partials are equal.

  (*Clairaut's Theorem* ??)

- If all 3rd order partials are continuous, the mixed 3rd order partials are equal.

- In general, if the n^{th} order partials are continuous, the mixed n^{th} order partials are equal.