Productivity

If $\mathcal{P} = f(\ell, k)$ gives the output $\mathcal{P}$ when ℓ units of labour and k units of capital are used, then $\mathcal{P}$ is called a **production function**.

$\dfrac{\partial \mathcal{P}}{\partial \ell}$ is defined to be the **marginal productivity w.r.t.** ℓ **(labour)**.

(rate of change in $\mathcal{P}$ w.r.t. ℓ when k is held fixed.)

$\dfrac{\partial \mathcal{P}}{\partial k}$ is defined to be the **marginal productivity w.r.t.** k **(capital)**.

(rate of change in $\mathcal{P}$ w.r.t. k when ℓ is held fixed.)

A **Cobb-Douglas production function** is a production function of the form

$$\mathcal{P} = C\, \boldsymbol{\ell}^{\alpha}\, \boldsymbol{k}^{\beta}$$

where C, α, β are constants with $\alpha + \beta = 1$.

Let A and B be two products such that a change in price for one affects the demand for the other. Hence the demand for each depends on the price of both. If q_A, q_B are the quantities demanded and p_A, p_B are the prices then

$$q_A = f(p_A, p_B)$$

$$q_B = g(p_A, p_B)$$

If p_A increases with p_B fixed, then q_A decreases $\implies \dfrac{\partial q_A}{\partial p_A} < 0.$

If p_B increases with p_A fixed, then q_B decreases $\implies \dfrac{\partial q_B}{\partial p_B} < 0.$

Definition:

If $\dfrac{\partial q_A}{\partial p_B} > 0$ and $\dfrac{\partial q_B}{\partial p_A} > 0$, then A and B are said to be **competitive products** or **substitutes**.

If $\dfrac{\partial q_A}{\partial p_B} < 0$ and $\dfrac{\partial q_B}{\partial p_A} < 0$, then A and B are said to be **complementary products**.