A **zero-row** of a matrix is a row consisting entirely of zeros.

A row that contains at least one nonzero entry is a **nonzero-row**.

The first nonzero entry in a nonzero-row is called the **leading entry**.

A matrix is said to be a **reduced matrix** provided that all of the following are true.

1. all zero-rows are at the bottom of the matrix

2. for each nonzero-row, the leading entry is 1, and all other entries in the column containing the leading entry are zeros.

3. the leading entry in each row is to the right of the leading entry in any row above it.

<u>Remark:</u> We can show that each matrix is equivalent to *exactly one* reduced matrix.

To solve a linear system we find the reduced matrix equivalent to the original augmented matrix.

The system of linear equations

$$\begin{aligned} A_{11}x_1 \; &+A_{12}x_2 \; +\cdots \; +A_{1n}x_n \; = \; C_1 \\ A_{21}x_1 \; &+A_{22}x_2 \; +\cdots \; +A_{2n}x_n \; = \; C_2 \\ &\vdots \\ A_{m1}x_1 \; &+A_{m2}x_2 \; +\cdots \; +A_{mn}x_n \; = \; C_m \end{aligned}$$

is called a **homogeneous system** if

$$C_1 = C_2 = \cdots = C_m = 0.$$

The system is a **nonhomogeneous system** if at least one of the C_i's is not equal to 0.

A homogeneous system,

$$\begin{aligned} A_{11}x_1 \; &+A_{12}x_2 \; +\cdots \; +A_{1n}x_n \; = \; 0 \\ A_{21}x_1 \; &+A_{22}x_2 \; +\cdots \; +A_{2n}x_n \; = \; 0 \\ &\vdots \\ A_{m1}x_1 \; &+A_{m2}x_2 \; +\cdots \; +A_{mn}x_n \; = \; 0 \end{aligned}$$

always has the solution

$$x_1 = 0, \, x_2 = 0, \; \cdots \; ,x_n = 0$$

which is called the **trivial solution**.

Theorem Let A be the reduced coefficient matrix of a homogeneous system of m linear equations in n unknowns. If A has exactly k nonzero-rows, then $k \leq n$. Moreover,

1. if $k < n$, the system has infinitely many solutions
2. if $k = n$, the system has a unique solution (the trivial solution).

Corollary A homogeneous system of linear equations with fewer equations than unknowns has infinitely many solutions.