A diagonal matrix of order n with $A_{ii} = 1$, $i = 1, 2, \cdots, n$ is called the **identity matrix** of order n.

A square matrix A is said to be an **upper triangular matrix** if all entries *below* the main diagonal are zero $\big(A_{ij} = 0$ for $i > j\big)$.

A square matrix A is said to be a **lower triangular matrix** if all entries *above* the main diagonal are zero $\big(A_{ij} = 0$ for $i < j\big)$.

Definition : If $A = \left[A_{i\,j} \right]$ and $B = \left[B_{i\,j} \right]$ are both $m \times n$ matrices, then the **sum** $A + B$ is the $m \times n$ matrix obtained by adding corresponding entries of A and B; that is, the ij^{th} entry of $A + B$ is $(A + B)_{ij} = A_{ij} + B_{ij}$.

If the size of A is different from the size of B, then $A + B$ is not defined.

Remark : If A is an $m \times n$ matrix with real number entries, we can write $A \in M_{m,n}(\mathbb{R})$.

Properties. If A, B, C, $\mathbf{0}$ have the same size

1. $A + B = B + A$ *commutative*

2. $A + (B + C) = (A + B) + C$ *associative*

3. $A + \mathbf{0} = \mathbf{0} + A = A$ *identity*

Definition : If A is an $m \times n$ matrix and k is a real number, then, by $k\,A$, we denote the $m \times n$ matrix obtained by multiplying each entry in A by k. This operation is called **scalar multiplication** and $k\,A$ is called a **scalar multiple** of A.

Properties. If A, B and $\mathbf{0}$ have the same size, then, for any scalars k and ℓ we have

1. $k\,(A + B) = k\,A + k\,B$
2. $(k + \ell)\,A = k\,A + \ell\,A$
3. $k\,(\ell\,A) = (k\,\ell)\,A$
4. $0\,A = \mathbf{0}$
5. $k\,\mathbf{0} = \mathbf{0}$.

Under the same conditions,

$$(A + B)^T = A^T + B^T$$

$$(k\,A)^T = k\,A^T$$

Subtraction

If A is any matrix, then the scalar multiple $(-1)\,A$ is simply written $-A$ and called the **negative** of A.

$$-A = (-1)\,A$$

Definition : If A and B are the same size, then, by $A - B$, we mean $A + (-B)$.

Definition : Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Then the product $A\,B$ is the $m \times p$ matrix C, whose typical entry $C_{i\,j}$ (row i, column j), is given by

$$C_{ij} = \sum_{k=1}^{n} A_{ik}\, B_{kj} = A_{i1}B_{1j} + A_{i2}B_{2j} + \;\cdots\; + A_{in}B_{nj} \;\; .$$

If the number of columns of A is **not equal** to the number of rows of B, the product is not defined.

Properties Let A, B, C be matrices and k be a scalar. Assuming that the sizes are compatible so that all products and sums are defined, we have

1. $A\,(B\,C) = (A\,B)\,C$ *associative*
2. $A\,(B + C) = A\,B + A\,C$

   $(A + B)\,C = A\,C + B\,C$ *distributive*
3. $k\,A\,B = k\,(A\,B) = (k\,A)\,B = A\,(k\,B)$
4. $(A\,B)^T = B^T\,A^T$