

A **linear function** in  $x$  and  $y$  has the form

$$P = a x + b y$$

where  $a$  and  $b$  are constants.

The problem of optimizing a linear function in two nonnegative variables, subject to constraints represented by a system of linear inequalities and linear equations, is called a **linear programming** problem in 2 variables.

The function to be maximized or minimized is called the **objective function**.

The (usually infinitely many) solutions to the system of constraints are called **feasible solutions** or **feasible points**. The collection of feasible solutions is the **feasible region**.

The solution(s) that maximizes or minimizes the value of the objective function is called the **optimum solution(s)**.

If a feasible region can be contained within a circle (of some finite radius) it is called a **bounded feasible region**; otherwise, it is **unbounded**.

If a feasible region contains at least one point it is said to be **nonempty**; otherwise, it is **empty**.

A **standard feasible region** is a bounded feasible region which includes all corners, edges and a nonempty interior.

\* **Fundamental Theorem of Linear Programming**

A linear function defined on a standard feasible region has a maximum (minimum) value and this value can be found at a corner.

\* If the feasible region is empty, there is no optimum solution.

\* If a feasible region is unbounded, and if the objective function has a maximum (or minimum) value, then that value will occur at a corner.

\* When an objective function attains its optimum value at more than one feasible point we say that

**multiple optimum solutions exist.**

If  $(a, b)$  and  $(c, d)$  are corner points at which an objective function is optimum, then the function will also be optimum at all points  $(x, y)$  where

$$x = (1 - t) a + t c$$

$$y = (1 - t) b + t d$$

and  $0 \leq t \leq 1$ .