University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA33 - Calculus for Management II

Examiner: S. Chopra　　　　　　　　　　Date: Aug 13th, 2010

Duration: 3hours

Provide the following information:

Surname (PRINT):____

Given Name(s) (PRINT):____

Student Number (PRINT):____

Signature:____

Read these instructions:

1. This examination paper has 14 numbered pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.

2. Put your solutions and/or rough work in the answer space provided. If you need extra space, use the back of a page or the blank page at the end of the exam.

3. You may use one standard hand-held calculator. All other electronic devices, extra paper, notes, and textbooks are forbidden at your workspace.

Please do not write in the box below:

Q1	/12
Q2	/8
Q3	/5
Q4	/10
Q5	/10
Q6	/6
Q7	/10
Q8	/9
Q9	/8
Q10	/10
Q11	/12
Total	/100

1. **[12 points]**

(a) Let A = $\begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 1 \\ 5 & 3 \\ 3 & 1 \end{bmatrix}$, C = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$. Find $A(B + 3C)^T$.

(b) Let the solution to part (a) be denoted as a new matrix D. Find the inverse of the matrix D using the method of cofactors.

2. **[8 points]** Solve the following non-homogeneous linear system using the row reduction method.

$$x_2 - 3x_3 = -5$$
$$2x_1 + 3x_2 - x_3 = 7$$
$$4x_1 + 5x_2 - 2x_3 = 10$$

3. **[5 points]** Let A and B be two 2×2 matrices where det A = -3 and B = $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$.
   Solve det $\left(\left((\det A^2)B\right)^{-1}\right)$.

4. **[10 points]** A produce grower is purchasing fertilizer containing three nutrients: A, B and C. The minimum weekly requirements are 80 units of A, 120 units of B, and 240 of C. There are two popular blends of fertilizer on the market. Blend I, costing \$8 a bag, contains 2 units of A, 6 of B, and 4 of C. Blend II, costing \$10 a bag contains 2 units of A, 2 of B, and 12 of C. How many bags of each blend should the grower buy each week to minimize the cost of meeting the nutrient requirements? (To earn full points, please include a neat labeled diagram and all calculations)

5. **[10 points]** Sketch on the same axis the level curves c = -1, 0, 1 for the functions

   (a) $f(x,y) = \left(\frac{x-y}{x}\right)$

   (b) $f(x,y) = \sqrt{x^2 + y^2 - 3}$

6. **[6 points]** Classify A and B as competitive, complementary, or neither if the demand equations for related products A and B are
$q_A = e^{-(p_A+3p_B)}$ and $q_B = \frac{8}{p_A^2 p_B^3}$
where q_A and q_B are the number of units of A and B demanded when the unit prices (in thousands of dollars) are p_A and p_B, respectively.

7. **[10 points]** If $xz + xyz^2 - x^3y^2z = 5$ determines z as a function of x and y, evaluate $\frac{\partial^2 z}{\partial y^2}$ when x = 1, y = 4 and z = 1.

8. **[9 points]** Let $z = e^{3x+4y}\sqrt{5x + 9y^2}$, where $x = rs^2$ and $y = 7rs^2$. Evaluate $\frac{\partial z}{\partial s}$ when r = 2 and s = -1.

9. **[8 points]** If $f(x,y) = x^3 - y^2 - 2x - 4$, find the critical prints of *f*. For each critical point of *f*, determine whether it corresponds to relative maximum, a relative minimum, or to neither, or whether no conclusion. (Hint: Use the second derivative test).

10. **[10 points]** Find the critical points for $f(x, y, z) = x^2 + y^2 - z^2$ subject to the constraints $x + y + z = 0$ and $x + y - z = 4$.

11. **[12 points]** Evaluate the integrals:

(a) $\int_{e}^{3}\int_{1}^{x} \ln y \ dy\, dx$

(b) $\int_{-1}^{0}\int_{1}^{x^2}\int_{x+y}^{2x+y} x \ dz\, dy\, dx$