University of Toronto Scarborough
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Term Test

MATA33H3 – Calculus for Management II

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Date: February 26, 2010
Duration: 110 minutes

1. **[6 points]** Let $A = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$.
   (a) Evaluate $D^{-1} + C^T B$.
   (b) Find $\det(A\,B + A\,C)$.
   (c) Find all possible diagonal matrices E which satisfy $E^2 = D$.

2. **[6 points]** Let A, B and C be 3×3 matrices with $\det A = 1$, $\det B = -2$ and $\det C = 3$. Evaluate the following
   (a) $\det\big((2\,A)\,B^2\big)$
   (b) $\det\big((\frac{1}{2}\,B)^{-1}\,C\big)$
   (c) $\det\big(A\,B\,C\big)^T$.

3. **[5 points]** Let $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 3 & 0 & 3 \end{bmatrix}$. Find all real numbers t such that the matrix $(A - t\,I)$ is **not** invertible.

4. **[18 points]** Find the maximum and minimum values of the function $Z = 3x + 4y$ subject to the constraints: $5x + 2y \le 40$, $3x + 3y \le 30$, $x + 2y \le 16$ and $x, y \ge 0$. (For full marks, your solution must include all of your calculations and some justification, along with a neat, labeled diagram of the feasible region clearly showing where Z is optimized.)

5. **[12 points]** Suppose a TV dealer has stores A and B and warehouses C and D. The cost of shipping a TV is \$18 from C to A, \$9 from C to B, \$24 from D to A and \$15 from D to B. Suppose that store A orders 25 TV sets and store B orders 30 TV sets. Suppose also that warehouse C has 45 TV sets and warehouse D has 40 TV sets available. Find the best way to fill these two orders so as to minimize cost, and find the minimum cost.

6. **[18 points]**

(a) Let $A = \begin{bmatrix} 1 & -3 & 2 & 0 \\ -2 & 6 & -3 & 1 \\ 2 & 0 & 2 & -1 \\ -1 & 2 & 0 & 3 \end{bmatrix}$. Use row reduction to determine if A is invertible. It it is, find A^{-1}. If it is not, explain why. Show all your work and indicate the row operations used.

(b) Solve the linear system:

$$\begin{array}{rcrcrcrcr} x & - & 3y & + & 2z & & & = & 1 \\ -2x & + & 6y & - & 3z & + & w & = & 0 \\ 2x & & & + & 2z & - & w & = & 1 \\ -x & + & 2y & & & + & 3w & = & 0 \end{array}.$$

7. **[15 points]** Use row reduction to solve the linear system:

$$\begin{array}{rcrcrcrcr} x & + & y & + & 2z & - & w & = & 4 \\ & & 3y & - & z & + & 4w & = & 2 \\ x & + & 2y & - & 3z & + & 5w & = & 0 \\ x & + & y & + & z & + & w & = & -3 \end{array}.$$

Show all your work and indicate the row operations used.

8. **[10 points]** Let $A = \begin{bmatrix} 2 & 0 & -2 \\ 3 & 1 & 2 \\ 1 & 0 & -3 \end{bmatrix}$.

(a) Without attempting to find the inverse, determine if A is invertible.

(b) If A is invertible, use the method of cofactors to find A^{-1}.

9. **[10 points]** Use Cramer's Rule to solve, if possible, the linear system, given in matrix form by

$$\begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & -2 \\ -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$