***** Sorry...no solutions are provided*****

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

Midterm Test

MATA33S - Calculus for Management II

Examiners: P. Glynn-Adey
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Date: February 13, 2017
Time: 5:00 pm
Duration: 110 minutes

Provide the following information

Last Name (PRINT BIG) _____

Given Name(s) (PRINT BIG) _____

Student Number _____

Signature _____

Circle your current Teaching Assistant and only your Tutorial Number

Bryan CHAN 18 22	Yuanyuan LI 21
Fazle CHOWDHURY 13 15 23 26	Ada (Jiawei) LIN 4
Kayleigh (Soomin) CHOY 16	Adrien NOURYA 9
Vera (Ruixue) DAI 6 19	Yang QIU 17
Rui GAO 1 2	Jacob (Sunghwan) YOO 11 25
Jianheng LI 14 20	Sheng ZANG 7 8

Read these Instructions

1. This test has 7 full questions 10 numbered pages. It is your responsibility to check at the beginning of the test that all of these pages are included.
2. Answer all questions in the work space provided. If you need extra space, use the back of a page or the bottom of Page 10. Clearly indicate the location of any continuing work.
3. Make your answers correct and complete. Show all of your work.
4. The following are forbidden at your workspace during any part of the test: calculators, smart phones, tablet devices, any kind of electronic transmission or receiving device, electronic dictionaries/translation devices, extra paper, textbooks, notes, opaque (i.e. non-see through) pen/pencil cases, hats/headcovers (except for religious reasons), or food. You may have one drink, but it cannot be in a paper cup or box.
5. You are encouraged to write your test in pen or other ink, not pencil. If any portion of your test is written in pencil, your entire test will be denied any re-grading privilege.

Do not write anything in the boxes below

Info	1	2	3	4	5	6	7	TOTAL
3	13	16	21	14	15	12	6	100

Instructions: Put your solutions in the answer spaces. Full-marks are awarded for answers that are correct, complete, and show a sufficient amount of relevant concepts from MATA33S.

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1. In all of this question let $A = \begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 11 & 4 \\ 3 & 1 \end{pmatrix}$, and $C = \begin{pmatrix} -5 & -2 \\ 9 & 6 \end{pmatrix}$.

   (a) Find $\left(AB - 3B^T\right)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. [5 points]

   (b) Evaluate $det(AB + CB)$. [4 points]

   (c) Find a matrix E such that $E^2 = C$ or show that there is no such matrix. [4 points]

2. In this question let $A = \begin{pmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{pmatrix}$.

(a) Find A^{-1}. [12 points]

(b) Express the matrix $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ as a sum of scalar multiples of the columns of A. [4 points]

3. (a) Find the maximum and minimum values (and all point(s) where they occur) of the objective function $Z = 4x+3y$ for the feasible region $\mathcal{R}$ that is given by the constraints:

$$-2 \leq x \leq 5, \qquad x - 2y \geq -10, \qquad x + 5y \leq 25, \qquad 2x - 5y \leq 10, \qquad x + y \geq -2.$$

(To earn full points, your solution must include a neat, labeled diagram of the feasible region $\mathcal{R}$, and all calculations/justifications. You may use space on the next page to continue your answer.) [17 points]

(b) Let S represent the set of points in the feasible region $\mathcal{R}$ in Part (a) for which the new objective function $W = -4x + 7y$ has constant value equal to 8. Accurately describe S in words. [4 points]

4. The two parts of this question are independent of each other.

(a) A company has a taxable income of $\$19,000,000$. The federal tax is 20% of that portion that is left after the provincial tax has been paid. The provincial tax is 25% of that portion that is left after the federal tax has been paid. Find the amount of the company's federal and provincial taxes. [7 points]

(b) Let a, b, and c be fixed real numbers and let x, y, and z be variables. Use Cramer's rule to solve for x in the linear system [7 points]

$$\begin{aligned} 2x + y + z &= a \\ -x + 0y + 2z &= b \\ 3x + y + 3z &= c \end{aligned}$$

(No method of solution other than Cramer's rule will earn any credit)

5. (a) Use the method of reduction to find the solution to the linear system

$$\begin{aligned} 2x_1 + 2x_2 - x_3 + 0x_4 + x_5 &= 11 \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= -7 \\ x_1 + x_2 - 2x_3 + 0x_4 - x_5 &= 1 \end{aligned}$$

[11 points]

(Be sure you state the reduced form of the augmented matrix.)

(b) Find the particular solution to the linear system in Part (a) that satisfies the conditions $x_1 + x_2 = -4$ and $x_1 + x_3 = -13$. [4 points]

6. Your financial company has two kinds of investment products: P_1 and P_2. These are sold to the public in units where one unit of P_1 costs \$600 and one unit of P_2 costs \$400. One can by any non-negative amount of units of P_1 or P_2.

   Each unit of P_1 contains 20 shares of Company A, 12 shares of Company B, and 40 shares of Company C. Each unit of P_2 contains 30 shares of Company A, 48 shares of Company B, and 20 shares of Company C.

   A particular investor states that she requires at least 600 shares of Company A, at least 240 shares of Company B, and at least 800 shares of Company C. How may units of P_1 and P_2 should the investor purchase so that her requirements above are satisfied and the total cost of her investment products is minimized? What is the least total cost? Provide a complete solution to this linear programming problem. You may assume there actually is a least total cost for the given requirements; you need not verify this. [12 points]

7. Let A be an $n \times n$ matrix where $n \geq 2$ and $A^2 = A$. Let c be a real number, $c \neq 1$. Find the inverse of the matrix $I - cA$. [6 points]

The remainder of this page can be used for rough work.