***** Solutions are posted as a separate document *****

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

Midterm Test

MATA33S - Calculus for Management II

Examiners: P. Glynn-Adey, R. Grinnell

Date: February 13, 2016
Time: 9:00 am
Duration: 2 hours

Provide the following information

Last Name (PRINT BIG) ________________

Given Name(s) (PRINT BIG) ________________

Student Number ________________

Signature ________________

Circle the name of your Teaching Assistant and Tutorial Number

Ayaan CHAUDHRY	26	Michael MOON	21 24
Ruixue DAI	6	Ke TONG	12
Taylor ESCH	13	Huiyi WANG	1
Rui GAO	5	Tianqi WANG	15
Terry (Yaodong) GAO	4	Dexter WU	16
Amir HEJAZI	25	Binya XU	23
Martin HO	7	Ruoqi YU	17
Pourya MEMARPANAHI	8	Elaine (Mengnan) ZHU	10

Read these Instructions

1. This test has 10 numbered pages. It is your responsibility to check at the beginning of the test that all of these pages are included.
2. Answer all questions in the work space provided. If you need extra space, use the back of a page or the last page, and indicate clearly the location of your continuing work.
3. Make your answers correct and complete and show all of your work.
4. The following are forbidden at your workspace during any part of the test: calculators, smart phones, tablet devices, any kind of electronic transmission or receiving device, electronic dictionaries, extra paper, textbooks, notes, opaque (i.e. non-see through) pen/pencil cases, or food. You may have one drink, but it cannot be in a paper cup or box.
5. You are encouraged to write your test in pen or other ink, not pencil. If any portion of your test is written in pencil, your entire test will be denied any re-grading privilege.

Do not write anything in the boxes below

Info	1	2	3	4	5	6	7	TOTAL
3	20	16	15	15	12	14	5	100

Instructions: Put your solutions and rough work in the answer spaces provided. Full-marks are awarded for answers that are correct, complete, and display a sufficient amount of relevant concepts from MATA33S.

---

1. In all of this question, let $B = \begin{pmatrix} 1 & 4 \\ -1 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} -1 & 17 \\ -3 & 16 \end{pmatrix}$.

   (a) Find $3B + C^T$ and $det\Big(det(B)C\Big)$. [3 + 4 marks]

   (b) Find the product BC and then express each column of BC as a sum of scalar multiples of the columns of B. [7 marks]

   (c) Find the matrix A such that $AB = C$. [6 marks]

2. (a) Use the method of reduction to find the solution to the linear system

$$\begin{aligned} -3x_1 - 18x_2 - 3x_3 + 0x_4 - 21x_5 &= 3 \\ 3x_1 + 18x_2 + 4x_3 + 1x_4 + 29x_5 &= 0 \\ 0x_1 + 0x_2 + 0x_3 - 2x_4 - 10x_5 &= -4 \end{aligned}$$

[8 marks]

(Be sure you state the reduced form of the augmented matrix.)

(b) Let k be a real constant and let $\heartsuit$ represent the system of three linear equations in variables x and y whose augmented matrix is $\left(\begin{array}{cc|c} 2 & 3 & 2 \\ 1 & 4 & 6 \\ 5 & k & 2 \end{array}\right)$. Find the value(s) of k so that $\heartsuit$ has a unique solution and then find the unique solution for each value of k.

[8 marks]

3. Find the maximum and minimum values (and all points(s) where they occur) of the objective function $Z = 6x + 2y$ for the feasible region $\mathcal{R}$ that is given by the five constraints:

$$x \geq -2, \qquad x - 6y \geq -14, \qquad 3x + y \leq 15, \qquad x - 8y \leq 5, \qquad y \geq -1$$

(To earn full points, your solution must include a neat, labeled diagram of the feasible region $\mathcal{R}$, and all calculations/justifications.)

[15 marks]

4. In all of this question, let $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -6 & -2 \\ 0 & 5 & 1 \end{pmatrix}$.

(a) Find all real values of x for which the matrix $A - xI$ is invertible where I is the 3×3 identity matrix. [7 marks]

(b) Use row reduction to find the inverse of the matrix $A - I$. [8 marks]

5. A student has money in three bank accounts that pay 5%, 7%, and 8% annual interest. She has three times as much invested in the 8% account as she does in the 5% account. Her total investment is $\$1,600$ and the total interest for the year is $\$115$. How much does she have invested in each account?

   (a) Set up the system of linear equations that represent the investment problem above. [4 marks]

   (b) Use the method of reduction to solve your system in Part (a) and state how much is invested in each account. [8 marks]

6. A small business produces two kinds of decorative wooden cabinets, A and B. Each unit of A requires 3 worker-hours to assemble and 30 minutes to paint. Each unit of B requires 2 worker-hours to assemble and 1 worker-hour to paint. There is a maximum of 240 worker-hours of assembly time per day and 80 worker-hours of painting time per day. Each unit of A has a fixed cost of \$100 and a revenue of \$600. Each unit of B has a fixed cost of \$80 and a revenue of \$480.

   (a) Use methods of linear programming to find how many of A and B should be made per day in order to maximize the total profit and satisfy all of the conditions above. What is the maximum profit? [10 marks]

(Part(b) is on the next page)

(b) Assume all of the conditions above plus the extra condition that the same number of A and B must be made per day. How many of A and B should be made per day in order to maximize the total profit now and what is the maximum profit? [4 marks]

7. Let A be an $n \times n$ matrix, $n \geq 3$. Assume $1R_1 + 2R_2 + 3R_3 + ... + nR_n = 0$ where R_i is the i^{th} row of A. Prove that A is not invertible. [5 marks]

(This page is intentionally left blank)