*** Sorry...No Solutions are Provided***

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA33 - Calculus for Management II

Examiners: R. Buchweitz
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Date: April 16, 2014
Time: 2:00 pm
Duration: 3 hours

Last Name (BIG PRINT): ____________________

Given Name(s) (BIG PRINT): ____________________

Student Number (BIG PRINT): ____________________

Signature: ____________________

Read these instructions:

1. This examination booklet has 12 pages. It is your responsibility to ensure at the beginning of the exam that all 12 pages are included.
2. Put your solutions and/or rough work in the answer spaces provided. If you need extra space, use the back of a page or blank page 12. Clearly indicate your continuing work.
3. The following are forbidden at your workspace: calculators, cell/smart phones, i-Pods/iPads, other electronic transmission/receiving devices, extra paper, notes, textbooks, pencil/pen cases, food, drink boxes/bottles with labels, backpacks, and coats/outdoor wear.
4. You may write your exam in pencil, pen, or other ink.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in these boxes:

A	1	2	3	4	5	6	7	8	**TOTAL**
30	13	16	17	14	14	15	14	17	150

Part A: 10 Multiple Choice Questions. Print the letter of the answer you think is most correct in the boxes on the first page. Each right answer earns 3 points. No answer/wrong/ambiguous answers earn 0 points.

1. Assume z is defined implicitly as a function of x and y by the equation $3z^2 = 2y + 12x$. The value of z_{xx} at the point $(1, 0, -2)$ is

   (A) $-\dfrac{1}{4}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) $-\dfrac{1}{2}$ (E) none of (A) - (D)

2. If the joint demand function for two products is $\alpha(x, y) = 100 - 4x + 85e^{-2y} - y^2$ and $\beta(x, y) = ln\left(\dfrac{1}{x}\right) + 5y - e^x$ where x and y are their respective prices, then the products are

   (A) competitive (B) complementary (C) both (A) and (B) (D) neither (A) nor (B)

3. If P is a 3×3 matrix such that $det(P) = -2$ then the value of $det\Big(det(P)\,(3P)^{-1}\Big)$ is

   (A) $\dfrac{4}{27}$ (B) 3 (C) $\dfrac{1}{3}$ (D) $-\dfrac{4}{27}$ (E) $\dfrac{4}{9}$ (F) none of (A) - (E)

4. The value of $\displaystyle\int_0^1 \int_2^4 x^2 y \, dy \, dx$ is

   (A) $\dfrac{28}{3}$ (B) $\dfrac{7}{3}$ (C) $\dfrac{2}{3}$ (D) 2 (E) 4 (F) none of (A) - (E)

5. The domain of the function $f(x, y) = \sqrt{y - x^2} + \dfrac{1}{x^2 - y^2}$ is all points (x, y) that are

   (A) on or above the curve $y = x^2$, but not $(0, 0)$
   (B) on or above the curve $y = x^2$, but not the points $(0, 0)$ nor $(\pm 1, 1)$
   (C) strictly above the curve $y = x^2$, but not the points $(0, 0)$ nor $(\pm 1, 1)$
   (D) none of (A) - (C)

6. If $z = (3x-4y)^3$ and $x = r+s^2$ and $y = 2r^2+s$, then $\dfrac{\partial z}{\partial s}$ evaluated at $r=1$ and $s=2$ is

(A) -48 (B) -24 (C) 9 (D) 24 (E) 12 (F) none of (A) - (E)

7. What values of the constant c will make $\begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$ and $\begin{pmatrix} 4 & -5 \\ 3 & c^2 \end{pmatrix}$ commute?

(A) $\pm\sqrt{2}$ (B) $\pm\sqrt{3}$ (C) ± 5 (D) ± 3 (E) $\pm\sqrt{5}$

8. The critical point of $h(x,y) = x^2+3y^2$ such that $x-3y=16$ is

(A) $(-4,4)$ (B) $(4,-4)$ (C) $(40,8)$ (D) $(-8,-8)$ (E) none of (A) - (D)

9. If the y-intercept of a certain level curve of $g(x,y) = xy^2+3x-y$ is -1, then the equation of that level curve is

(A) $x = \dfrac{y+1}{y^2+3}$ (B) $x = \dfrac{-y-1}{y^2+3}$ (C) $x+y=-1$ (D) none of (A) - (C)

10. Let A be an $n \times n$ matrix, $n \geq 2$. How many of the following five properties are equivalent to A being invertible ? (i) $det(A) \neq 0$ (ii) $AX = 0$ has the trivial solution (iii) $A + A$ is invertible (iv) $AX = B$ has a solution for any $n \times 1$ matrix B (v) $AK = KA$ for some $n \times n$ matrix K

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

<u>BE SURE YOU HAVE PUT YOUR ANSWERS IN THE 1-ST PAGE BOXES</u>

Part B: 8 Full Solution Questions. Put your solutions and rough work in the answer space provided. Full points are awarded only if your solutions are correct, complete, and sufficiently display appropriate, relevant concepts from MATA33.

1. The production function (i.e output) for a very small home-based graphics business is given by $f(x, y) = 20x + 26y - x^2 - 3y^2 + 9,000$ where x and y are the quantities of two inputs. The unit prices of the inputs are \$2 and \$4, respectively. Use the method of Lagrange Multipliers to find the value of x and y that maximizes the output if the budget is \$280. (You may assume that x and y are non-negative real numbers, all of the budget is used for the two inputs, and that the constrained critical point obtained does correspond to the maximum output).

[13 points]

2. The parts of this question are independent of each other.

(a) Assume z is defined implicitly as a function of variables x and y by the equation $e^{xz} = xyz$. Find the value of y so that the point $w = (1, y, -1)$ satisfies this equation and then evaluate z_y at w.

[7 points]

(b) Assume the function $z = h(x, y) = ax^2 + by^2 + c^2xy - 4x - 8y$ has a critical point at $(x, y) = (0, 1)$ and that the second-derivative test is inconclusive at this point. Find the value(s) of the constants a, b, and c. [9 points]

3. In all of this question assume the sales volume of a new product (in thousands of units) is given by $S = \dfrac{AT + 450}{\sqrt{A + T^2}}$ where T is the time (in months) since the product was first introduced and A is the amount (in hundreds of dollars) spent each month on advertising. Assume $A, T > 0$.

   (a) Calculate the partial derivative of S with respect to time. Use that partial derivative to predict the number of months that will elapse before sales volume begins to decrease if the amount allocated to advertising is held fixed at $\$9,000$ per month. [9 points]

   (b) Calculate the appropriate second-order partial derivative to determine whether the marginal sales volume with respect to time is increasing or decreasing as a function of advertising expenditure. (Note: we assume $A, T > 0$ and we do not assume the fixed monthly advertising expenditure as in part (a). [8 points]

4. The parts of this question are independent of each other.

(a) Let $z = g(x, y) = xe^{xy}$ and let $G(x, y) = \dfrac{\partial z}{\partial x} + \dfrac{\partial^2 z}{\partial y \partial x}$. Find the function $y = u(x)$ such that $G(x, u(x)) = 0$ for all x in the domain of u. [8 points]

(b) Assume $z = h(ar + bt, cr - kt)$ where $h(x, y)$ is a MATA33 function for which the chain rule is valid and a, b, c, and k are constants such that $ak + bc = 0$.

Prove that $k\dfrac{\partial z}{\partial r} + c\dfrac{\partial z}{\partial t} = 0.$ [6 points]

5. Assume the following in all of this question. A company manufactures and sells two products (X and Y) each day. Both products require the same two tasks for their manufacturing: assembly and painting. Each unit of X produced and sold requires 3 worker-hours of labour to assemble and each unit of Y produced and sold requires 2 worker-hours to assemble. For painting, X requires $1/2$ hour and Y requires 1 hour, per unit. We also have the requirement that the number of units of Y made and sold per day must be at least as large as the number of units of X made and sold per day. There is a total of 240 worker-hours available per day for assembly and 80 worker-hours available per day for painting.

   Suppose the revenue for each unit of X made and sold per day is \$60 and for each unit of Y made and sold per day is \$40. Set-up a linear programming problem to find the whole number of units of X and Y made and sold per day in order to maximize the total revenue subject to the conditions implied above. Begin by letting x represent the number of X made and sold per day and y represent the number of Y made and sold per day. In your calculations, assume x and y are non-negative real numbers. Draw an accurate feasible region and show all appropriate details. [14 points]

6. (a) Evaluate $\int_0^4 \int_0^2 \left(x + \sqrt{2y+1}\right) dx\, dy$ [7 points]

(b) Evaluate $\int_0^1 \int_{e^x}^{e} \frac{y}{ln(y)}\, dy\, dx$ [8 points]

7. Find all critical points of the function $h(x, y, z) = y^3 + yx^2 - 3y^2 + z^2 + 2x^2$. For each one, determine whether it yields a relative maximum, minimum, or a saddle point. [14 points]

8. The parts of this question are independent of each other.

(a) Consider the system of equations in variables x, y, and z where α is a real constant:

$$\begin{aligned} 3x + 3y + z &= 1 \\ 4x + \alpha y + 2z &= 2 \\ 2\alpha x + 2\alpha y + \alpha z &= 0 \end{aligned}$$

Use determinants to find all values of α for which the system has a unique solution. Then use Cramer's rule to find z for each such unique solution. [11 points]

(b) Assume A is an $n \times n$ matrix ($n \geq 3$) and has the property that each entry in the first row equals the sum of the $n - 1$ entries in the column below it. Prove that $det(A) = 0$. [6 points]

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