***** Sorry...No Solutions are Provided*****

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA33S - Calculus for Management II

Examiners: P. Glynn-Adey
R. Grinnell

Date: April 21, 2017
Time: 9:00 am
Duration: 170 minutes

Last Name (PRINT BIG) ______________________________

Given Name(s) (PRINT BIG) ______________________________

Student Number (PRINT BIG) ______________________________

Signature ______________________________

Read these instructions

1. This examination has 12 pages. Check this number at the beginning of the exam.
2. Put your solutions, answers, and rough work in the answer spaces provided for each question. Show all of your work. If you need extra space, use the back of a page or blank page 12 and clearly indicate your continuing work.
3. The following are forbidden at your workspace or with you during any part of the exam:
   calculators, laptop computers, smart phones, cell phones, smart watches, iPods/iPads, other electronic transmission/receiving devices, extra paper, notes, textbooks, opaque pencil/pen cases, food, drink boxes/bottles with labels, backpacks, and coats/outdoor wear. You cannot wear a hat or head cover, except for reasons of religion/creed.
4. You may write your exam in pencil, pen, or other ink. You may have an eraser or correction fluid and a ruler at your workspace.

Do not write anything in these boxes

1	2	3	4	5	6	7	8	9	10	**TOTAL**
17	14	17	16	15	14	15	11	14	17	150

Instructions: Put your solutions, answers, and rough work in the answer space provided. Show all of your work. Full points are awarded only if your solutions/answers are correct, complete, and sufficiently display appropriate, relevant concepts from MATA33S.

1. In each part of this question let $f(x,y) = \dfrac{\sqrt{x}}{\sqrt{y^2 - x + 1}}$.

   (a) Let D represent the domain of the function f. State D using set-bracket notation (i.e. $D = \{\,(x,y)\,|\,\ldots\}$) and appropriate inequalities. Carefully sketch D and lightly shade all points that are **not in** D. [7 points]

   (b) Find the function $x = h(y)$ that gives the level curve of $f(x,y) = 1$. [4 points]

   (c) Let $z = \Big(f(x,y)\Big)^2$. Show that $\dfrac{\partial z}{\partial x} = (y^2+1)\left(\dfrac{z}{x}\right)^2$. [6 points]

2. The two parts of this question are independent of each other.

(a) Let x and y represent numbers of units of products **X** and **Y** sold, respectively. Assume the joint-revenue function is given by $R(x, y) = \dfrac{500xy}{2x + 3y}$. Show that when equal amounts of **X** and **Y** are sold, the sum of the marginal revenue functions is a constant.

[7 points]

(b) Let $a, b < -1$ and $c < b$. Assume (a, b, c) be a critical point of a MATA33 function $f(x, y, z)$. Assume the Hessian matrix is $H\Big(f(a, b, c)\Big) = \begin{pmatrix} ab & b & 0 \\ a & ab & 0 \\ 0 & 0 & -c^3 \end{pmatrix}$.

Determine whether f has a relative maximum, relative minimum, or saddle point at the critical point (a, b, c).

[7 points]

3. The two parts of this question are independent of each other.

(a) Let $w = 2x^2\sqrt{y+5}$ where $x = 2r^3 + 4s^2$ and $y = (r+6)^{2/3}s$. Use the Chain Rule to evaluate the partial derivative $\dfrac{\partial w}{\partial r}$ when $s = 1$ and $r = 2$. Express your answer as a simplified rational number. [9 points]

(b) Assume the equation $4z^3 + x^2z^2 = 4xy$ defines z implicitly as a function of variables x and y. Show that the point $(x, y, z) = (4, 6, 2)$ satisfies this equation and then evaluate $\dfrac{\partial z}{\partial x}$ at this point. Express your answer as a simplified rational number. [8 points]

4. Find all critical points of the function $f(x, y, z) = x^3 + x^2 + y^2 + z^2 - xy + xz$ and classify each point as a relative maximum, relative minimum, or a saddle point. [16 points]

5. In all parts of this question assume A and C are 5×5 matrices such that the entries in A are integers and $det(A) = -1$ and $det(C) = 2$.

(a) Find the following: (i) $det(-2A^2C)$ and (ii) $det\Big(det(C^{-1})\, C^T\Big)$ [3 + 3 points]

(b) Let P be the matrix that is obtained by multiplying Row 1 of C by 2 and Row 4 of C by -3. Find $det(P)$. [3 points]

(c) Assume B is a 5×1 matrix whose entries are integers. Show that the matrix equation $AX = B$ has a unique solution and that this solution has only integer entries.

[6 points]

6. Your company builds above ground swimming pools. A customer wants a pool with a flat circular bottom. The material for the bottom costs $\$1/m^2$ and the material for the re-enforced sides costs $\$5/m^2$. **Use the Lagrange Multiplier method** to find the radius and height (r, h) which minimizes the total material cost of a pool with volume of $100\pi\ m^3$. You may assume the total material cost has a minimum value and that the Lagrange Multiplier method yields the minimizing value for r and h. [14 points]

   (**If you solve this problem using any method other than Lagrange Multipliers, your solution will earn at most** 4 **points**).

7. The parts of this question are independent of each other.

(a) Find the values of the constants a, b, and c so that the surface $z = x^2 + axy + by^2 + c$ contains the points $(1, 1, 11)$, $(2, 1, 16)$, and $(1, 2, 22)$. [7 points]

(b) Recall that $F = \left(1 + \frac{r}{x}\right)^x - 1$ is the effective rate formula where $r > 0$ is the annual percentage rate and $x > 0$ is the number of annual compounds of interest.

Show that $F_x = (F + 1)\left[\frac{ln(F+1)}{x} - \frac{r}{x+r}\right]$. [8 points]

8. Let $f(x,y) = xy + \frac{27}{x} + \frac{8}{y}$. This function f has a unique critical point. Find this critical point and determine whether it gives a local maximum, minimum, or saddle point.

[11 points]

9. (a) Evaluate $\int_1^2 \int_0^4 (x + 2y)\, dx\, dy$. [5 points]

(b) Let T be the triangle with vertices $(0, 0)$, $(0, 2)$, and $(2, 2)$.
Evaluate $\int\int_T y^2 e^{xy} dA$. (Hint: sketch the triangle and choose an appropriate order of integration.) [9 points]

10. A company has 4 locations L_1, L_2, L_3, L_4 and sells 8 of the same products $X_1, X_2, \ldots, X_8$ at each location. Let $P = [P_{i,j}]$ be the **profit matrix** where

$P_{i,j} =$ the company's profit (in \$1,000's) when exactly one unit of product X_j is sold at location L_i.

(a) If the company's profit from selling 7 units of product X_2 at location L_4 is \$84,000, what is the value of the entry $P_{4,2}$? [3 points]

(b) State the matrix Q such that $A = QP$ where $A = [A_{1,j}]$ and $A_{1,j}$ is the company's average profit in dollars from selling exactly one unit of product X_j at all of the 4 locations of the company. [4 points]

(c) State the matrices E and C so that the single entry in the matrix product EPC^T is the total profit in dollars from selling exactly j units of product X_j at all of the 4 locations of the company. [3 + 3 points]

(d) State the matrices E and F such that the single entry in the matrix product EPF is the sum of all entries in the matrix P. [4 points]

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