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University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA33S - Calculus for Management II

Examiners: P. Glynn-Adey
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Date: April 8, 2016
Time: 2:00 pm
Duration: 170 minutes

Last Name (PRINT BIG) ______________________________

Given Name(s) (PRINT BIG) ______________________________

Student Number (PRINT BIG) ______________________________

Signature ______________________________

Read these instructions

1. This examination has 12 pages. Check this number at the beginning of the exam.
2. Put your solutions, answers, and rough work in the answer spaces provided for each question. If you need extra space, use the back of a page or blank page 12 and clearly indicate your continuing work.
3. The following are forbidden at your workspace or with you during any part of the exam:

   calculators, laptop computers, smart phones, cell phones, smart watches, iPods/iPads, other electronic transmission/receiving devices, extra paper, notes, textbooks, opaque pencil/pen cases, food, drink boxes/bottles with labels, backpacks, and coats/outdoor wear. You cannot wear a hat or headcover, except for reasons of religion/creed.
4. You may write your exam in pencil, pen, or other ink. You may have an eraser or correction fluid and a ruler at your workspace.

Do not write anything in these boxes

1	2	3	4	5	6	7	8	9	10	**TOTAL**
18	13	14	16	16	14	16	10	15	18	150

Instructions: Put your solutions, answers, and rough work in the answer space provided. Show all of your work. Full points are awarded only if your solutions/answers are correct, complete, and sufficiently display appropriate, relevant concepts from MATA33S.

1. In all of this question let $f(x,y) = \sqrt{-4x^2 - 2y + 16}$.

   (a) Let D represent the domain of the function f. State D using set-bracket notation (i.e. $D = \{\,(x,y) \mid \ldots\}$) and an appropriate inequality, and carefully sketch/shade D.

   [6 points]

   (b) Find the function $y = h(x)$ that gives the level curve of f passing through $(1,4)$ and sketch this function h on the axis you used in Part (a). [5 points]

   (c) Evaluate the function $\left[\Big(f_x(x,y) + f_y(x,y)\Big)f(x,y)\right]^2$ at the point $(-1,0)$. [7 points]

2. Use the method Lagrange Multipliers to find the critical point of the function

   $w = f(x, y, z) = x \ln(x) + y \ln(y) + z \ln(z)$ subject to the constraint $x + y + z = 6$.

   You may assume the function f has a maximum or minimum at this critical point. Determine which one of these is correct. [13 points]

3. The parts of this question are independent of each other.

(a) Let $z = 2x^2y^2 - 2x + 4y$ where $x = (r+3)\sqrt{s}$ and $y = e^{(s-1)}r^3$. Use the Chain Rule to evaluate $\dfrac{\partial z}{\partial s}$ when $s = 1$ and $r = -2$. [7 points]

(b) Assume the function $h(x, y) = ax^2 - by^2 - cxy + 4x - y$ has a critical point at $(1, 0)$ and the second-derivative test is inconclusive at this point. Find a, b, and c. [7 points]

4. Find all critical points for the function $f(x, y, z) = x^3 + xz^2 - 3x^2 + y^2 + 2z^2$ and classify each point as a relative maximum, relative minimum, or a saddle point. [16 points]

5. In all of this question assume the equation $2z^2 = x^2 + 2xy + xz$ defines z implicitly as a function of the independent variables x and y.

(a) Let $x = -4$ and $y = 0$ in the equation above and solve for z. [4 points]

(b) Calculate $\dfrac{\partial^2 z}{\partial x \partial y}$. [8 points]

(c) Evaluate the mixed partial derivative you found in Part (b) at each of the points obtained through Part (a). [4 points]

6. In all of this question consider the system of three linear equations in the variables x, y, and z, and real parameter t:

$$\begin{aligned} tx - y + z &= 0 \\ 6x + y - 2z &= 2 \\ t^2x - 2y - z &= 1 \end{aligned}$$

(a) Use a determinant to find all values of t such that the system above has a unique solution. [6 points]

(b) Use Cramer's rule to solve for x and y in the system above for the values of t you determined in Part (a). Do not solve for z. [8 points]

7. The parts of this question are independent of each other.

(a) Let $x, y > 0$ be the unit prices of products X and Y, respectively. Let $P = e^{-(\alpha x + \beta y)} + \dfrac{x}{y}$ and $Q = 8x^{-\alpha}y^{-\beta}$ be the quantity (i.e. demand) functions for X and Y, respectively. Here α and β are constants. Show that if $\alpha, \beta > 0$, then X and Y are complementary. [7 points]

(b) The following is an equation in mathematics of finance: $r_L = r + D\dfrac{\partial r}{\partial D} + \dfrac{dC}{dD}$ where $r =$ the bank deposit rate, $r_L =$ the bank earning rate, $C =$ administrative costs, and $D =$ bank deposit level. If $\sigma = \left(\dfrac{r}{D}\right)\left(\dfrac{\partial r}{\partial D}\right)^{-1}$, show that $r_L = r\left[\dfrac{\sigma + 1}{\sigma}\right] + \dfrac{dC}{dD}$. [5 points]

(c) Make a good sketch of the surface $y^2 + z^2 = 4$ in $\mathbf{R}^3$. Clearly label your axes and provide at least four labeled points on the surface. [4 points]

8. Let $\mathcal{R}$ be the feasible region defined by the five inequalities:

$$x \geq 0, \quad y \geq 0, \quad 2x + y \leq 4, \quad -x + y \leq 1, \quad 2x + 3y \geq 12.$$

Let $Z = 3x + 2y$. Determine the maximum and minimum values of Z and where they occur for the feasible region $\mathcal{R}$, or determine that Z has no optimal values on $\mathcal{R}$. Sufficiently justify your answer. [10 points]

9. (a) Economists have determined that a person's status in Canadian society is given by the function $S(b,n) = \int_0^b \int_0^n \left(2x + \sqrt{y}\right) \, dx \, dy$ where b is their age in decades and n is their annual income in units of \$10,000. Find the status of a person of age 40 with an annual income of \$90,000. [7 points]

(b) Evaluate $\int_0^2 \int_{e^y}^{e^2} \frac{x^2}{\ln x} \, dx \, dy$. (Hint: sketch the region of integration) [8 points]

10. A company has three locations L_1, L_2, L_3 and sells five of the same products P_1, P_2, P_3, P_4, P_5 at each location. Let $R = [R_{i,j}]$ be the **revenue matrix** where

$R_{i,j} =$ the company's revenue (in \$100) from selling one unit of product P_j at location L_i.

(a) If the company's revenue from selling 8 units of P_3 at location L_2 is \$3,200, what is the value of the entry $R_{2,3}$? [3 points]

(b) State the matrix B so that $A = RB^T$ where $A = [A_{i,1}]$ and $A_{i,1}$ is the company's average revenue in dollars from selling exactly one unit of each product at location i.

[4 points]

(c) State the matrices E and C so that the single entry in the matrix product ERC^T is the total revenue in 1000's of dollars from selling exactly $(j+2)$ units of product P_j at each location of the company. [3 + 3 points]

(d) Suppose the company has the following new projection for its sales: For each location, the revenue of P_1 increases by \$40 per unit; the revenue for P_3 decreases by 20% per unit; and the revenue for P_5 doubles per unit. The unit revenues for P_2 and P_4 do not change at any location. State the matrix S so that $S + R$ gives the new sales revenue for all products and all locations. [5 points]

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