***** Sorry...solutions will not be provided *****

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA33 - Calculus for Management II

Examiners: R. Buchweitz
R. Grinnell

Date: April 22, 2015
Time: 9:00 am
Duration: 3 hours

Last Name (PRINT BIG) ______________________________

Given Name(s) (PRINT BIG) ______________________________

Student Number (PRINT BIG) ______________________________

Signature ______________________________

Read these instructions

1. This examination has 12 pages. Check this number at the beginning of the exam.
2. Put your solutions, rough work, etc. in the answer spaces provided. If you need extra space, use the back of a page or blank page 12. Clearly indicate your continuing work.
3. The following are forbidden at your workspace: calculators, cell/smart phones, smart watches, i-Pods/iPads, other electronic transmission/receiving devices, extra paper, notes, textbooks, opaque pencil/pen cases, food, drink boxes/bottles with labels, backpacks, and coats/outdoor wear. You cannot wear a hat or headcover, except for reasons of religion/creed.
4. You may write your exam in pencil, pen, or other ink. You may have an eraser or correction fluid and a ruler.

Print letters for the Multiple Choice Questions in these boxes

1	2	3	4	5	6	7

Do not write anything in these boxes

A	1	2	3	4	5	6	7	8	**TOTAL**
28	15	15	18	18	12	16	13	15	150

Part A: 7 Multiple Choice Questions. Print the letter of the answer you think is most correct in the boxes on the first page. Each right answer earns 4 points. No answer/wrong/ambiguous answers earn 0 points.

1. If $A = \begin{pmatrix} 5 & 8 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 0 & 6 \end{pmatrix}$ then $det(A^{-1} + B^T)$ equals

   (A) -12 (B) -38 (C) -39 (D) -33 (E) none of (A) - (D)

2. The value of $\displaystyle\int_1^e \int_1^4 \frac{\sqrt{y}}{x}\, dy\, dx$ is

   (A) $\dfrac{14e}{3}$ (B) $\dfrac{14}{3}$ (C) $2\sqrt{3}$ (D) $\dfrac{7}{3}$ (E) none of (A) - (D)

3. If $w = y^2 e^{4xy}$ and $y \neq 0$, then $\dfrac{\partial w}{\partial y}$ equals

   (A) $\dfrac{2w}{y}(1+xy)$ (B) $2w(1+2xy)$ (C) $\dfrac{w}{y}(1+2xy)$ (D) $\dfrac{2w}{y}(1+2x)$ (E) $\dfrac{2w}{y}(1+2xy)$

4. Let $a > b > 0$ and $Z = ax - by$. If $\mathcal{R}$ is the feasible region given by $0 \le x \le y$ and $y \ge 1$, then which one of the following is true?

(A) Z has both a maximum and minimum on $\mathcal{R}$.

(B) Z has a maximum on $\mathcal{R}$, but not a minimum.

(C) Z has a minimum on $\mathcal{R}$, but not a maximum.

(D) Z has neither a maximum nor a minimum on $\mathcal{R}$.

5. If $z = (3x - 4y)^3$ and $x = r + s^2$ and $y = 2r^2 + s$, then $\dfrac{\partial z}{\partial r}$ evaluated at $r = 1$ and $s = 2$ is

(A) 39 (B) 46 (C) -46 (D) -41 (E) -39 (F) none of (A) - (E)

6. The critical point of $h(x, y) = 8x^2 + y^2 + 5$ subject to the constraint $8x - 2y = 12$ is

(A) $(\frac{12}{7}, \frac{6}{7})$ (B) $(-1, -10)$ (C) $(3, 6)$ (D) $(-2, -14)$ (E) $(1, -2)$

7. Let A and B be $n \times n$ invertible matrices for some $n > 1$. How many of the following five statements must be true?

(i) $A + B$ is invertible. (ii) $A^{-1}B$ is invertible.

(iii) $(AB)^{-1} = B^{-1}A^{-1}$ (iv) $B^T AB$ is invertible.

(v) there exists a matrix C such that ABC is the 3×3 identity matrix.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5

BE SURE YOU HAVE PUT YOUR ANSWERS IN THE 1-ST PAGE BOXES

Part B: 8 Full Solution Questions. Put your solutions and rough work in the answer space provided. Show all of your work. Full points are awarded only if your solutions are correct, complete, and sufficiently display appropriate, relevant concepts from MATA33.

1. In all of this question let $f(x, y) = \sqrt{x^2 - 4y + 8}$.

   (a) Let D represent the domain of the function $g(x, y) = f(x, y) + ln(y + 1)$. State D using set-bracket notation (i.e. $D = \{ (x, y) | \dots \}$) and appropriate inequalities, and carefully sketch/shade D. [6 points]

   (b) Find the function $y = h(x)$ that gives the level curve of f passing through $(2, 1)$. [4 points]

   (c) Assume $f(x, y) = f(y, x)$ and $x \neq y$. Show that $y = -x - 4$ where $x \neq -2$. [5 points]

2. The parts of this question are independent of each other.

(a) For each real number t, let $S(t)$ represent the system of linear equations in variables x, y, and z:

$$\begin{aligned} tx - y + z &= 0 \\ 6x + y - 2z &= 2 \\ tx - 2y - z &= 1 \end{aligned}$$

Find all values of t for which $S(t)$ has a unique solution. Then use Cramer's rule to express y in terms of each of these values t. [7 points]

(b) Let r and α be positive variables and let $E = \left(1 + \frac{r}{\alpha}\right)^{\alpha} - 1$ be the "effective rate function". Prove that $\frac{\partial E}{\partial \alpha} = (E+1)\left[ln\left(1+\frac{r}{\alpha}\right) - \frac{r}{\alpha + r}\right]$. [8 points]

3. In all of this question let $C = \dfrac{3xy}{8x + 2y}$ represent the total joint manufacturing cost (in millions of Canadian dollars) of two products, **X** and **Y**. Assume $x > 0$ is the number of hundreds of units of **X** produced and $y > 0$ is the number of hundreds of units of **Y** produced.

(a) Find and simplify the marginal cost functions. [6 points]

(b) Show that when the same number of **X** and **Y** is produced, then the sum of the marginal cost functions is a constant. [4 points]

(c) Assume the marginal cost functions are equal when the sum of the production numbers of **X** and **Y** is $1,200$ units in total. Calculate the numbers of units of X and Y produced in this case and find the corresponding total joint manufacturing cost. [8 points]

4. Find the critical point(s) for the following two functions and classify each point as a relative maximum, relative minimum, or a saddle point.

(a) $f(x, y) = 4x^2 e^y - 2x^4 - e^{4y}$ [11 points]

(b) $g(x, y, z) = x^2 + y^2 + yz + xz$ [7 points]

5. In all of this question assume the equation $xz^2 + y^2z = 14$ defines z implicitly as a function of independent variables x and y.

(a) Show that $\dfrac{\partial z}{\partial y} = \dfrac{-2yz}{2xz + y^2}$. [4 points]

(b) Calculate the exact value of $\dfrac{\partial^2 z}{\partial x \partial y}(3, -1, 2)$ and sufficiently simplify your answer. You may assume that $\dfrac{\partial z}{\partial x}(3, -1, 2) = -\dfrac{4}{13}$. [8 points]

6. (a) Evaluate $\int_0^1 \int_0^1 \Big(3+x\Big) e^x \, dx \, dy$. [7 points]

(b) Evaluate $\int\int_{\mathcal{R}} x\sqrt{1+y^3} \; dA$ where $\mathcal{R}$ is the triangular region with vertices $(0,0)$, $(0,2)$, and $(2,2)$. [9 points]

7. A joint production function is given by $P = kx^a y^b$ where $x > 0$ is the number of units of labour, $y > 0$ is the number of units of raw material, and a, b, and k are positive constants with $a + b = 1$. Each unit of labour costs \$$m$ and each unit of raw material costs \$$n$. There is a budget of \$$T$ which is the sum of the total labour cost and total raw material cost.

   Use the Lagrange Multiplier method to find x and y (in terms of a, b, k, m, n, and T) that maximizes P under the assumption that all of the budget \$$T$ is used. You need not find the actual maximum production value. You may assume that the constrained critical point obtained via the Lagrange Multiplier method actually does correspond to a maximum value for P. A solution not using the Lagrange Multiplier method will earn little credit.

   [13 points]

8. ECC (Expensive Condominiums of Canada) is a large real estate developer that builds and sells condominiums in 4 Canadian cities: C_1, C_2, C_3, C_4. There are 3 models of condos M_1, M_2, M_3 that are sold in each city. Let $R = [R_{i,j}]$ be the 3×4 "revenue matrix" where

$R_{i,j} =$ ECC's revenue (in \$1,000) from selling each unit of condo model M_i in city C_j.

(a) If ECC's revenue from selling one model M_2 in city C_3 is \$653,000, what is the value of the entry $R_{2,3}$? [2 points]

(b) State the matrices A and B such that: (i) the entries in the product AR give, for each city, the average revenue in \$1,000 over the 3 condo models and (ii) the entries in the product RB give, for each model, the the total revenue in dollars over the 4 cities.

[5 points]

(c) In city C_1, ECC builds and hopes to sell x units of model M_2 and y units of model M_3. The construction cost for each unit of M_2 is 33% of its revenue in C_1 and the construction cost for each unit of M_3 is 37% of its revenue in C_1. ECC can spend at most \$7 million to construct all of these two condo types (M_2 and M_3) in city C_1 and desires a profit of at least \$12 million for selling these two condo types in C_1. Unfortunately, in city C_1 it is possible that perhaps none of M_2 or M_3 actually sells.

Give a system of 4 linear inequalities that describe this situation.

Remember that Profit = Revenue - Cost. [8 points]

(This page is intentionally left blank)