University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

***** Solutions are not provided*****

MATA33 - Calculus for Management II

Examiners: R. Grinnell
E. Moore

Date: April 19, 2013
Time: 9:00 am
Duration: 3 hours

Last Name (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number (PRINT): ______________________________

Signature: ______________________________

Read these instructions:

1. It is your responsibility to ensure that all 13 pages of this examination are included.
2. Put your solutions and/or rough work in the answer spaces provided. If you need extra space, use the back of a page or blank page 13. Clearly indicate your continuing work.
3. Show all work and justify your answers. Full points are awarded for solutions that are correct, complete, and show appropriate concepts from MATA33.
4. You may use one standard calculator that does not perform any: graphing, matrix operations, numerical/symbolic differentiation or integration. Cell/smart phones, i-Pods/Pads, other electronic transmission/receiving devices, extra paper, notes, textbooks, pencil/pen cases, food, drink boxes/bottles with labels, and backpacks are forbidden at your workspace.
5. You may write in pencil, pen, or other ink.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in these boxes

A	1	2	3	4	5	6	7	8	**TOTAL**
30	17	14	15	16	16	11	15	16	150

Part A: Ten Multiple Choice Questions **Print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 3 points and no answer/wrong answers earn 0 points.

1. If $z = 8y^2 ln(y^2 - 3x) - \frac{4}{y}$ then $\left.\frac{\partial z}{\partial y}\right|_{(1,2)}$ equals (A) $\frac{83}{3}$ (B) 257 (C) 79 (D) 80 (E) 129 (F) none of (A) - (E)

2. If the y-intercept of a certain level curve of $g(x,y) = xy^2 + 3x - y$ is -1, then the equation of that level curve is

   (A) $x = \frac{y+1}{y^2+3}$ (B) $x = \frac{-y-1}{y^2+3}$ (C) $x + y = -1$ (D) none of (A) - (C)

3. If A is a 3×3 matrix such that $det(A) = -2$ then $det\left[\left(det(A^2)\right)(2A)^{-1}\right]$ equals

   (A) -4 (B) -2 (C) -1 (D) 1 (E) 2 (F) 4

4. Let M be an $n \times n$ matrix where $n \geq 2$. Exactly how many of the following five mathematical properties imply that M is invertible ?

(i) $det(M) > 0$ (ii) $MX = 0$ has the trivial solution (iii) $M^2 \neq 0$

(iv) $MX = B$ has a solution for any $n \times 1$ matrix B

(v) $CM^T = I$ for some 3×3 matrix C

(A) 4 (B) 3 (C) 2 (D) 1 (E) 5

5. If R is the feasible region defined by the inequality $0 \leq \frac{1}{2}x \leq y \leq x + 1$ then for what values of the constant α does the function $Z = \alpha x + y$ not have a maximum value on R ?

(A) -1 (B) $\frac{-1}{2}$ (C) 0 (D) (A) and (B) (E) (B) and (C) (F) (A) and (C)

6. If $z = \dfrac{x + e^y}{y}$ where $x = rs^2 + se^r$ and $y = 8r + r^2$ then $\dfrac{\partial z}{\partial s}$ at $(r, s) = (1, 4)$ is

(A) $\dfrac{2+e}{9}$ (B) $\dfrac{8+e}{4}$ (C) $\dfrac{8+e}{9}$ (D) $\dfrac{6+e}{9}$ (E) none of (A) - (D)

7. If the equation $z^2 = 16x - 7y$ defines z implicitly as a function of independent variables x and y, then the value of z_{xx} when $x = 1$, $y = 0$ and $z = -4$ is

(A) -2 (B) -1 (C) 0 (D) 1 (E) a number not in (A) - (D)

8. The value of $\int_0^4 \int_0^1 \left(3\sqrt{x} + 2y\right)\ dy\ dx$ is

(A) 20 (B) $\frac{16}{3}$ (C) 16 (D) $\frac{26}{3}$ (E) 12 (F) none of (A) - (E)

9. The critical points of $f(x, y) = 3x + y + 6$ subject to the constraint $x^2 + y^2 = 9$ must satisfy the equation (A) $x = y$ (B) $x = 2y$ (C) $x = -2y$ (D) $x = 3y$ (E) $y = 2x$ (F) $x = -3y$ (G) none of (A) - (F)

10. For what values of the constant a will the function $f(x, y) = ax^2 - \frac{1}{2}y^2 + xy - x + y$ have a relative minimum at the critical point $(0, 1)$?

(A) $a > 0$ (B) $a > \frac{1}{2}$ (C) $a < \frac{-1}{2}$ (D) $a < 0$ (E) (A) or (C)

(F) real values of a not precisely described by any of (A) - (E) (G) no values of a

BE SURE YOU HAVE PUT YOUR ANSWERS IN THE 1-ST PAGE BOXES

Part B: Eight Full Solution Questions Full points are awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA33.

1. In all of this question let $h(x,y) = \dfrac{\sqrt{9-x^2-y^2}}{y^2-4} + \dfrac{1}{xy}$

   (a) Give an accurate, labeled sketch of the domain D of h. Use light shading to clearly indicate D. Use solid lines for portions of the boundary included in D, and dashed lines or small circles for portions not included in D. [6 points]

   (b) Find $h_x(2,1)$ and state your answer as a rational number, not a decimal. [5 points]

   (c) Use set-bracket notation to describe all solutions to the equation $h(x,y) = h(y,x)$. [6 points]

2. The parts of this question are independent of each other.

(a) Let X and Y represent products where the unit prices are \$$x$ and and \$$y$, respectively. Assume the demand functions are $f(x,y) = e^{-(x+y)} + \dfrac{4}{y}$ and $g(x,y) = \dfrac{8}{x^2y} + \dfrac{1}{x}$, respectively. Determine whether X and Y are complementary, competitive, or neither.

[6 points]

(b) Let $z = h(x,y)$ be a typical "MATA33 function" (i.e. all second partial derivatives are continuous) where $x = r + s$ and $y = r - s$.

Show that $\dfrac{\partial^2 z}{\partial s \partial r} = \dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial y^2}$. [8 points]

3. The parts of this question are independent of each other.

(a) Let $A = \begin{pmatrix} 1 & 3 & 5 & 0 \\ 3 & 4 & 2 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$. Find $det(A)$. Determine (with justification) whether there exists a matrix C such that $C^2 = A$. [9 points]

(b) Sketch and label the feasible region R determined by the three inequalities: $y \geq 0, \quad x + 2y \leq 8, \quad 4x + 5y \geq 40$. Optimize the function $Z = 20x + 30y$ on R or, show that Z has no optimum values on R. [6 points]

4. Find the critical points of the function $f(x, y, z) = 4xy - x^4 - y^4 - 3z^2$. For each critical point determine whether it yields a local maximum, local minimum, or is a saddle point.

[16 points]

5. A company manufactures two products, A and B. The selling price of A is \$$x$ per unit and the selling price of B is \$$y$ per unit. It costs \$6 to manufacture one unit of A and \$20 to manufacture one unit of B. The demand (i.e. quantity) functions are $f(x, y) = 5(y - x)$ units of A and $g(x, y) = 500 + 5(x - 2y)$ units of B. Find the selling prices of A and B that maximize the total profit and verify that these prices actually do maximize the total profit. (Recall that Total Profit = Total Revenue - Total Manufacturing Cost). [16 points]

6. In all of this question let $I = \int_1^3 \int_0^{ln(x)} x \, dy \, dx$

(a) Give an accurate, labeled sketch of the region $\mathcal{D}$ over which the integration in I takes place. [3 points]

(b) Evaluate I by first reversing the order of integration. State your answer in terms of familiar mathematical constants, not decimals. [8 points]

7. A large rectangular box of volume 48 cubic metres has a bottom, left and right sides, front and back faces, but no top. The bottom material costs \$4 per square metre, the front and back face material costs \$2 per square metre, and the left and right side material costs \$1 per square metre. Use the technique of Lagrange multipliers to find the dimensions of the such a box where the total cost of material is least. You may assume the constrained critical point does yield a minimum value of the total cost. State your final answers in terms of familiar mathematical constants, not decimals. [15 points]

8. A town has three coffee shops: S_1, S_2 and S_3. Each shop sells a 355 ml cup of coffee of the same four flavours: F_1 (Columbian), F_2 (Imperial), F_3 (Mocha) and F_4 (Vanilla).

   Assume there is a matrix $A = [a_{i,j}]$ where $a_{i,j} =$ (the price in cents of a cup of coffee of flavour j at coffee shop i between 9am and 10am on Friday, April 19, 2013). Assume $a_{i,j}$ is a positive integer.

   (a) State the matrices B and C such that:

   (i) the entries in the product AB^T give, for each coffee shop, the average price in dollars of a cup of coffee over the four flavours. [3 points]

   (ii) the entries in the product CA give, for each flavour, the average price in dollars of a cup of coffee over the three coffee shops. [3 points]

   (b) Suppose at 11am that: all coffee prices at S_1 increase by 10 cents; all coffee prices at S_2 decrease by 5%; and at S_3, Columbian and Mocha coffees increase by 10%, while the other two flavours at S_3 do not change. State the matrix N such that the entries in $A + N$ give these revised coffee cup prices at 11am in cents. [5 points].

   (c) Assume from 9am to 10am that coffee shop S_i sells $u_{i,j}$ cups of coffee of flavour F_j. Consider the matrix $U = [u_{i,j}]$ where all entries are non-negative integers. Let R_i be the total revenue in dollars of selling coffee of the four flavours at shop S_i during the hour above. Describe (using matrix notation and appropriate terminology) how we can find R_i using the matrices A and U. [5 points]

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