University of Toronto Scarborough
Department of Computer & Mathematical Sciences

Final Examination

MATA33H3 – Calculus for Management II

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Duration: 3 hours

1. **[11 points]** Two chemical plants, plant A and plant B, produce three types of fertilizer: LP, MP and HP. At each plant, the fertilizer is produced in a single production run, so the three types are produced in fixed proportions. Plant A produces 1 ton of LP, 2 tons of MP and 3 tons of HP in a single production run, and charges \$600 for what was produced in one run. Plant B produces 1 ton of LP, 5 tons of MP and 1 ton of HP, and charges \$1000 for what was produced in one run. If a customer needs 100 tons of LP, 260 tons of MP and 180 tons of HP, how many production runs should be ordered from each plant to minimize costs? What is the minimal cost?

   (To earn full points, your solution must include a neat labelled diagram and show all calculations and appropriate justification.)

2. **[8 points]** Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 2 & -3 \\ -1 & -3 & 5 \end{bmatrix}$. Use row reduction to determine if A is invertible. If it is, find A^{-1}. If it is not, explain why. Show all your work and indicate the row operations used.

3. **[7 points]** Let $A = \begin{bmatrix} 2 & 5 & -3 & -2 \\ -2 & -3 & 2 & -5 \\ 1 & 3 & -2 & 2 \\ -1 & -6 & 4 & 3 \end{bmatrix}$. Evaluate $\det A$. Show all your work and indicate the operations used.

4. **[6 points]** Without attempting to solve the system of equations,

$$\begin{array}{rcrcrcl} 2x & & & + & z & = & 0 \\ 4x & + & 2y & - & 3z & = & 0 \\ 5x & + & 3y & + & z & = & 0 \end{array},$$

   show mathematically that $x = 0$, $y = 0$, $z = 0$ is the only solution.

5. **[9 points]** Let $f(x, y) = \dfrac{\sqrt{y}}{\sqrt{y^2 - x + 1}}$.

   (a) Let D be the domain of f.

      i. Describe D in set notation using appropriate inequalities.

      ii. Draw a sketch that clearly shows D.

   (b) Write the equation of the typical level curve of f corresponding to $f(x, y) = c$, in the form $x = g(y)$, if possible. What is the range of possible values for c?

6. **[6 points]**

   (a) Suppose z is a function of x and y, where both x and y are functions of s and t. State a Chain rule that expresses $\dfrac{\partial z}{\partial t}$ in terms of the derivatives of these functions.

   (b) Let $z = 2\,x^2 \, \ln|3x - 5y|$, where $x = s\,\sqrt{t^2+2}$ and $y = t - 3\,e^{2-s}$. Use part (a) to evaluate $\dfrac{\partial z}{\partial t}$ when $s = 1$ and $t = 0$.

7. **[6 points]** If $x\,z^2 - y^2\,z = 1$ determines z as a function of x and y, find $\dfrac{\partial^2 z}{\partial x^2}$.

8. **[12 points]** Let $A = \begin{bmatrix} x & 0 & 1 \\ y & y & z \\ x & -z & 2 \end{bmatrix}$.

   (a) Evaluate $\det A$.

   (b) Put $f(x,y,z) = \det A$ and find the critical points of f.

   (c) What information, if any, does the second derivative test give you about the critical points you found in part (b)?

9. **[9 points]** Let $f(x,y,z) = x^3 + x^2 + y^2 + z^2 - xy + xz$. Find the critical points of f. Use the second derivative test to determine whether each critical point is a relative (local) maximum, a relative (local) minimum or neither.

10. **[8 points]** Find the extrema of $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraints $z = x^2 + y^2$ and $x + y + z = 12$.

    (You may assume that the extrema occur at (constrained) critical points.)

11. **[8 points]**

    (a) A coffee retailer sells two premium coffee beans, Mocha and Kona. If Mocha beans are priced at x dollars for a 500 gm bag and Kona beans at y dollars for a 500 gm bag, market research says that $35-8x+4y$ bags of Mocha and $40+9x-7y$ bags of Kona will be sold. The wholesale cost is \$4 per kilogram for Mocha and \$8 per kilogram for Kona. How should the retailer price the premium coffee beans to maximize her profit. Justify that these prices do maximize profit.

    (b) Determine if the coffee beans in part (a) are competitive products, complementary products or neither.

12. **[10 points]** Evaluate the following integrals.

    (a) $\displaystyle\int_{\ln 2}^{2} \int_0^x e^x \, dy \, dx.$

    (b) $\displaystyle\int_{-1}^{1} \int_0^{1+x^2} \int_0^{1+x^2+y} dz \, dy \, dx.$