University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA33 - Calculus for Management II

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Date: April 22, 2008
Duration: 3 hours

Provide the following information:

Surname (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number (PRINT): ______________________________

Signature: ______________________________

Read these instructions:

1. This examination paper has 14 numbered pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.
2. Put your solutions and/or rough work in the answer space provided. If you need extra space, use the back of a page or the blank page at the end of the exam.
3. You may use one standard hand-held calculator. All other electronic devices, extra paper, notes, and textbooks are forbidden at your workspace.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	8	9	10	**TOTAL**
45	7	10	10	10	7	7	12	12	16	14	150

Part A: Multiple Choice Questions For each of the following **print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 4.5 points and no answer/wrong answers earn 0 points. Justification is neither required nor rewarded, but a small workspace is provided for your calculations.

1. If $f(x,y) = \dfrac{x+y}{x-y}$ then $f_x(2,1)$ equals

   (a) -2 (b) 2 (c) 1 (d) 0

2. At the point $(1,1)$ the function $f(x,y) = x^4 + y^4$ has

   (a) a relative minimum (b) a relative maximum (c) a saddle point (d) none of the above

3. Assume A and B are 2×2 matrices such that $A^3 - AB^2 = 0$. Then

   (a) none of (b), (c), or (d) need be true

   (b) $A = B$ or $A = -B$ or $A = 0$ (c) $A^2 = B^2$ (d) $AB = BA$

4. $\int_1^2 \int_3^6 x^2 dy dx$ is equal to (a) $\frac{15}{4}$ (b) 7 (c) $\frac{45}{4}$ (d) 63

5. The domain of the function $f(x,y) = (1 - x^2 - y^2)^{-1/2}$ is exactly

(a) the set of points that are strictly outside the unit circle

(b) the set of points that lie on or outside the unit circle

(c) the set of points that lie on or inside the unit circle

(d) the set of points that are strictly inside the unit circle

6. A general system of m linear equations in n unknowns where $n > m > 1$

(a) has at least one solution

(b) has infinitely many solutions

(c) has a unique solution

(d) may not have any solutions

7. If $z = x^2 e^{xy}$ and x and y are independent variables then $\frac{\partial z}{\partial x}(x,y)$ equals

(a) $e^{xy}(2 + xy)$

(b) $xe^{xy}(2 + xy)$

(c) $2xye^{xy}$

(d) $xe^{xy}(2 + x^2)$

8. The equation of the level curve of $h(x, y) = \dfrac{y}{x^2+1} - 2$ that passes through $(2, 5)$ is

(a) $y = -1$ (b) $y = -x^2 - 1$ (c) $y = x^2 + 1$ (d) none of the above

9. The joint demand functions for two products **X** and **Y** are given by $f(x, y) = 20 - x - e^{-y}$ and $g(x, y) = 50 + \sqrt{x} + 3y$ where x and y are the unit prices of **X** and **Y**, respectively. We may conclude that the products are

(a) complementary (b) competitive (c) both (a) and (b) (d) neither (a) nor (b)

10. Let R be the feasible region consisting of all points in and on the sides of the triangle with corners $(0, 0)$, $(2, 0)$, and $(1, 1)$. If $z = ax + by$ where a and b are non-zero constants then we may conclude that

(a) z has a maximum at a corner point

(b) there exist values of a and b for which z has a maximum at every point on some side of R

(c) the minimum value of z occurs at $(0, 0)$

(d) only (a) and (b) are true

(e) each of (a), (b), and (c) is true

(Did you print the letters of the correct answers in the boxes on the first page ?)

Part B: Full-Solution Questions Write clear and neat solutions in the answer spaces provided. Show all of your work. Full points are awarded for solutions only if they are correct, complete, and sufficiently display relevant concepts from MATA33.

1. Use the method of Lagrange multipliers to find the maximum value of $f(x, y) = xy + 2x$ subject to the constraint $2x + y = 30$ (You may assume that the critical point obtained does correspond to a maximum). [7 points]

2. A joint cost function is given by $c(x, y) = ln(xy^2 + 1) + \dfrac{6x^2}{y}$ where x and y represent the numbers of units of two products.

   (a) Find the marginal cost functions at $(4, 3)$ rounded to 2 decimal places. [7 points]

   (b) State a mathematical relationship that involves the cost function at $(4, 3)$ and $(5, 3)$, and a marginal cost function at $(4, 3)$ [3 points]

3. Let $f(x,y) = 3xe^y - x^3 - e^{3y}$. Find the critical point(s) of f. For each one, use the second derivative test to determine whether it corresponds to a relative maximum, minimum, or a saddle point. State the maximum/minimum value(s) of f, should they occur. [10 points]

4. Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 3 & -6 \end{bmatrix}$ Find all real values of x for which the matrix $B(x) = A - xI$ is **not** invertible.

[10 points]

5. Let $z = \dfrac{x}{y}$ where $x = se^t$ and $y = 1 + se^{-t}$. Use the chain rule to find $\dfrac{\partial z}{\partial s}$ when $s = 2$ and $t = 0$ [7 points]

6. Assume the equation $z^3 + 2x^2z^2 = xy$ defines z implicitly as a function of independent variables x and y. Find z_x when $x = z = 1$ [7 points]

7. A company makes two products, **A** and **B**, whose selling prices per unit are $\$x$ and $\$y$, respectively. Their fixed production costs are $\$m$ and $\$n$ per unit, respectively. The demand for **A** is $f(x, y) = 5(y - x)$ and the demand for **B** is $g(x, y) = 500 + 5(x - 2y)$. Find the selling prices that maximize the total profit and verify that these prices actually do maximize profit. (A useful concept is that Total Profit = Total Revenue - Total Cost.)

[12 points]

8. Let a and b be constants such that $0 < b < a$. Use techniques from linear programming to find the value of a and b so that the objective function $z = ax + by$ has a maximum value of 46 and a minimum value of 14 when (x, y) is subject to the constraints $3 \leq x \leq 9$, $x + 3y \geq 6$, $x - 3y \geq -6$, and $x, y \geq 0$

[12 points]

9. (a) Evaluate $I = \int_1^e \int_0^x ln(x)\ dydx$ [9 points]

(Question 9 continued)

(b) Evaluate $J = \int\int_R y^2 e^{xy}\, dA$ where R is the triangular region with vertices $(0,0)$, $(1,1)$, and $(0,1)$. [7 points]

10. A rectangular box with no top has a volume of $60m^3$. The material for the front and bottom costs $\$5/m^2$ and the material for the back, and left and right sides costs $\$1/m^2$. Use the method of Lagrange multipliers to find the dimensions of the box whose material cost is smallest (You may assume that the critical point produced actually does result in a least material cost). [14 points]

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