University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA33 - Calculus for Management II

*** Sorry...no solutions will be posted ***

Examiner: R. Grinnell

Date: August 15, 2016
Time: 2:00 pm
Duration: 170 minutes

Provide the following information

Lastname (Print) ______________________________

Given Name(s) (Print) ______________________________

Student Number ______________________________

Signature ______________________________

Instructions

1. This exam has 15 numbered pages. It is your responsibility to check at the beginning of the exam that all of these pages are included.
2. Answer all of the Part B (Full Solution) questions in the work space provided. If you need extra space, use the back of a page or the last page and clearly indicate the location of your continuing work.
3. Full points are awarded for Part B (Full Solution) questions only if they are correct, complete, clear, and sufficiently display concepts and methods of MATA33. Show all work.
4. The following are forbidden at your exam writing space by accident or intent: all calculators, cell phones, smart phones, any electronic device (e.g. translation device, tablet device), scrap paper, text books, study notes, food, drinks (except in a metal or clear plastic bottle), opaque pen/pencil cases, bags.
5. You cannot wear a hat/head covering except for religious reasons.
6. You may write in pencil, pen, or other ink, and you may use correction tape/fluid.

Print letters for Part A questions in these boxes

1	2	3	4	5	6	7	8	9	10	11	12

Do not write anything in the boxes below

Part A
30

Part B

1	2	3	4	5	6	7	8
18	14	14	14	13	17	14	16

Total
150

Part A (Multiple Choice and True/False Questions) For the following six Multiple Choice questions, **clearly and unambiguously print the letter of the answer you think is most correct in the box at the top of page 2**. Each right answer earns 3 points. No answer or wrong answers earn 0 points. Justification is neither required nor rewarded.

1. If $A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 4 \\ 2 & -3 \end{pmatrix}$ then $det\Big(2(A^{-1})B^T\Big)$ equals

   (A) 12 (B) 6 (C) -6 (D) 20 (E) none of (A) - (D)

2. If $f(x, y) = \dfrac{x^2 y}{y + 2x}$ then the value of $f_x(1, 2)$ is

   (A) 3/4 (B) 5/4 (C) 16/25 (D) 3 (E) none of (A) - (D)

3. The critical point of $f(x, y) = 3x^2 + 2y^2 + 6$ subject to the constraint $x + y = 5$ is

   (A) $(2, 3)$ (B) $(-2, 7)$ (C) $(1, 4)$ (D) $(3, 2)$ (E) $(4, 1)$

4. Products A and B have unit prices $x > 0$ and $y > 0$, respectively. Their demand functions are $\alpha(x, y) = e^{(-2x+3y)} + \dfrac{y}{x}$ and $\beta(x, y) = \dfrac{5y^{-1/2}}{\sqrt{x}}$, respectively. We may conclude that A and B are

(A) competitive (B) complementary (C) neither competitive nor complementary

5. If $z = 3x^2 + 2xy$ and $x = r^2 + s$ and $y = \dfrac{s}{r+1}$, then the value of $\dfrac{\partial z}{\partial r}$ when $r = 1$ and $s = 2$ is

(A) 19 (B) 34 (C) 43 (D) $\dfrac{123}{2}$ (E) 37 (F) none of (A) - (E)

6. Let (a, b, c) be a critical point of a MATA33 function $w = f(x, y, z)$ where $a, b < -1$. Assume the Hessian matrix is $A = Hf(a, b, c) = \begin{bmatrix} ab & b & 0 \\ a & ab & 0 \\ 0 & 0 & -a \end{bmatrix}$ At the point (a, b, c) we may conclude that

(A) f has a relative maximum (B) f has a relative minimum

(C) f has a saddle point (D) the second-derivative test gives no information

Part A Continued For the following six True/False questions, answer **T** if and only if the given assertion is always true. Otherwise answer **F**. **Clearly and unambiguously print your letters in the answer boxes at the top of page 2**. Correct answers earn 2 points and incorrect/blank answers earn 0 points.

7. If $f(x,y)$ is a MATA33 function and $f_{xx}(a,b)f_{yy}(a,b) > [f_{xy}(a,b)]^2$ and $f_{xx}(a,b) > 0$, then f has a relative minimum at (a,b).

8. A nonzero linear objective function defined on a bounded standard feasible region has both a maximum value and a minimum value.

9. If $f(x,y)$ and $g(x,y)$ are MATA33 functions and $f(x,y)$ subject to the constraint $g(x,y) = 0$ has a critical point (a,b), then $f_x(a,b) = f_y(a,b) = 0$.

10. Let A be a 3×3 matrix and K be a 3×1 matrix. Assume all entries in A and K are integers and $|det(A)| = 1$. We may conclude that Cramer's rule shows the unique solution to the matrix equation $AX = K$ has only integer entries.

11. If $h(x,y)$ is a polynomial function in the variables x and y, then $h_{xyx}(x,y) = h_{yxx}(x,y)$.

12. If $f(x,y)$ is a MATA33 function and $I = \int_0^4 \int_0^{\sqrt{y}} f(x,y)\,dx\,dy$, then the correct expression for I with the order of integration reversed is $I = \int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx$.

(Be sure you have printed the Part A answers in the boxes on Page 2)

Part B (Full Solution Questions) Put your solutions/answers in the spaces provided. Full marks are awarded if and only if your solutions/answers are correct, complete, clear and sufficiently demonstrate relevant ideas, methods, and concepts from MATA33. Show all work.

1. In all of this question let $f(x, y) = \sqrt{x^2 - 2y + 4}$.

   (a) Find the level curve of f that passes through $(8, -3)$ and write it in the form $y = g(x)$.
   [4 points]

   (b) Let $\mathcal{S}$ represent the domain of the function $h(x, y) = f(x, y) + \sqrt{x} + \sqrt{4 - x} + \sqrt{y - 2}$. Describe $\mathcal{S}$ using set notation and appropriate inequalities. Make a small sketch of $\mathcal{S}$. Use shading to indicate $\mathcal{S}$, clearly show its boundary lines, and label all corner points.
   [8 points]

   (c) Show that $f(x, y)\Big(f_x(x, y) - f_y(x, y)\Big) = x + 1$ for all (x, y) where the left side of this equation is defined.
   [6 points]

2. In all of this question assume the equation $yz^2 + x^2z = 14$ defines z implicitly as a function of independent variables x and y.

   (a) Verify that $z_x = \dfrac{-2xz}{2yz + x^2}$. [4 points]

   (b) Evaluate z_{xy} at the point $(-1, 3, 2)$ and simplify your final answer as much as possible. You may assume that $z_y(-1, 3, 2) = -\dfrac{4}{13}$. [10 points]

3. A rectangular box has a base, four sides, no top, and a volume of $V > 0$. Find the dimensions of such a box that has the smallest possible surface area. Your answers should be simple expressions in terms of V. Show all work and justify that the minimum surface area is obtained. [14 points]

4. The parts of this question are independent of each other.

(a) Let $z = x^2 + xy^3 + 2ln(x)$ where $x = ut^2 + s^3$ and $y = u + te^s$. Use the chain rule from MATA33 to find $\dfrac{\partial z}{\partial t}$ and then evaluate it when $u = 2,\ t = 1,\ s = 0$. [7 points]

(b) Let $w(r, s) = f(r^2 - s^2, s^2 - r^2)$ where f is a MATA33 function for which the chain rule is valid. Show that $s\dfrac{\partial w}{\partial r} + r\dfrac{\partial w}{\partial s} = 0$. [7 points]

5. Find the critical point(s) of the function $f(x, y, z) = x^2z + y^2x + z^2 - 2y$. Classify each as a relative minimum, relative maximum, or a saddle point. Sufficiently justify your answer.

[13 points]

6. In all of this question let $A = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$ and let I be the 3×3 identity matrix. The two parts of this question are related to each other.

(a) Use the method of reduction to solve the matrix equation $AX = 3IX$ where X is a 3×1 variable matrix. You need not show ERO notation. [6 points]

(b) Find all values of the real variable x for which the matrix $A - xI$ is not invertible. Show all work and sufficiently justify your answer. Part (a) is helpful. [11 points]

Question 6 answer space continues on the next page.

Question 6 continued.

7. In all of this question let $\mathcal{R}$ represent the triangular region in the plane whose corners are $(-2,-1)$, $(0,0)$, and $(0,-1)$.

   (a) Draw a small picture of $\mathcal{R}$ and give its set notation description (with appropriate inequalities) using

   (i) Vertical sections (i.e. y-simple) [4 points]

   (ii) Horizontal sections (i.e. x-simple) [3 points]

   (b) Evaluate $\displaystyle\int\int_{\mathcal{R}} e^{y^2}\, dA$. Your final answer should be expressed in terms of familiar mathematical constants. Choose the order of integration wisely: the function $f(t) = e^{t^2}$ does not have an antiderivative.

   [7 points]

8. A small company produces widgets using two positive real "input quantities" x and y. The unit price of x and y is \$4 and \$8, respectively. The widget production function is given by $p(x,y) = 12x + 20y - x^2 - 2y^2$. Use the Lagrange multiplier method to determine the maximum total cost and minimum total cost to produce exactly 74 widgets. State the input quantities that give these costs. Show all work and sufficiently justify your answer.

[16 points]

(This page is intentionally left blank)