*** Sorry...No Solutions Will be Provided ***

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA33 - Calculus for Management II

Examiner: R. Grinnell

Date: August 13, 2014
Time: 9:00 am
Duration: 3 hours

Last Name (PRINT BIG): ______________________

Given Name(s) (PRINT BIG): ______________________

Student Number (PRINT BIG): ______________________

Signature: ______________________

Read these instructions:

1. This examination has 12 pages. It is your responsibility to ensure at the beginning of the exam that all 12 pages are included.
2. Put your solutions and/or rough work in the answer spaces provided. If you need extra space, use the back of a page or blank page 12. Clearly indicate your continuing work.
3. The following are forbidden at your workspace: calculators, cell/smart phones, i-Pods/iPads, other electronic transmission/receiving devices, extra paper, notes, textbooks, pencil/pen cases, food, drink boxes/bottles with labels, backpacks, and coats/outdoor wear.
4. You may write your exam in pencil, pen, or other ink.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in these boxes:

A	1	2	3	4	5	6	7	8	**TOTAL**
35	15	17	10	7	15	16	19	16	150

Part A: 10 Multiple Choice Questions. Print the letter of the answer you think is most correct in the boxes on the first page. Each right answer earns **3.5** points. No answer/wrong/ambiguous answers earn 0 points.

1. If A is a 3×3 matrix such that $det(A) = -3$, then the value of $det\left[2det(A^T)(6A)^{-1}\right]$ is

   (A) $-\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $-\frac{1}{3}$ (D) $-\frac{2}{3}$ (E) none of (A) - (D)

2. If $z = \dfrac{ln(4x^2+5)}{2y}$ then the value of $\dfrac{\partial z}{\partial x}$ when $x = y = 1$ is

   (A) -4 (B) $\frac{4}{9}$ (C) $-\frac{4}{9}$ (D) $\frac{16}{9}$ (E) none of (A) - (D)

3. At the point $(1,1)$ the function $g(x,y) = x^3 + e^y - ey$ has

   (A) a local maximum (B) a local minimum (C) a saddle point (D) none of (A) - (C)

4. The demand functions for two products A and B are given by $q_A = 2^{-(3x+y)} + 5$ and $q_B = -12x^{-1.5}y^{-2} + 18$, respectively, where x and y are the unit prices of A and B, respectively. We may conclude that the products are

   (A) competitive (B) complementary (C) neither competitive nor complementary

5. The value of $\int_1^4 \int_0^{y-1} 4x \, dx \, dy$ is

(A) 9 (B) 28/3 (C) 14 (D) 17 (E) 18 (F) none of (A) - (E)

6. The domain of the function $f(x,y) = \sqrt{y - x^2} + \dfrac{1}{x^2 - y^2}$ is all points (x, y) that are

(A) on or above the curve $y = x^2$, but not $(0, 0)$

(B) on or above the curve $y = x^2$, but not the points $(0, 0)$ nor $(\pm 1, 1)$

(C) strictly above the curve $y = x^2$, but not the points $(0, 0)$ nor $(\pm 1, 1)$

(D) none of (A) - (C)

7. Assume the equation $z^2 = 13x - y$ defines z implicitly as a function of independent variables x and y only when $z < 0$. The value(s) of $\dfrac{\partial z}{\partial x}$ when $x = 4$ and $y = 3$ is (are)

(A) $-\dfrac{13}{14}$ (B) $\dfrac{13}{14}$ (C) $\dfrac{11}{7}$ (D) $-\dfrac{11}{7}$ (E) (A) and (B) (F) (C) and (D)

(G) none of (A) - (F)

8. Assume $z = ln(x^2 - y)$ where $x = r^2 s$ and $y = r - 3s^2$.

The value of the partial derivative $\frac{\partial z}{\partial s}$ when $r = 1$ and $s = 2$ is

(A) $\frac{14}{15}$ (B) 1 (C) $\frac{16}{15}$ (D) $-\frac{8}{15}$ (E) $-\frac{2}{3}$ (F) none of (A) - (E)

9. The constrained critical point and associated Lagrange multiplier of the function $f(x, y) = 8x - 5y$ satisfying the constraint $4x^2 + 5y = 19$ is

(A) $(-1, 3),\ \lambda = 1$ (B) $(-1, 3),\ \lambda = -1$ (C) $(-1, 10),\ \lambda = 1$

(D) $(1, 3),\ \lambda = -1$ (E) $(1, -3),\ \lambda = 1$ (F) $(1, 3),\ \lambda = 1$

10. Let $\mathcal{R}$ be the feasible region consisting of all points in and on the triangle whose vertices are $(0, 0)$, $(2, 0)$, and $(1, 1)$. Let a and b be negative constants and consider the objective function $Z = ax + by$. We may conclude the following about Z and $\mathcal{R}$:

(A) Z has a maximum value at some corner point of $\mathcal{R}$.

(B) the minimum value of Z occurs at $(0, 0)$.

(C) there exist values of a and b such that Z has a minimum value at every point on some edge of $\mathcal{R}$.

(D) (A) and (C) are both true, but (B) is false.

(E) (A) and (B) are both true, but (C) is false.

BE SURE YOU HAVE PUT YOUR ANSWERS IN THE 1-ST PAGE BOXES

Part B: 8 Full Solution Questions. Put your solutions and rough work in the answer space provided. Full points are awarded only if your solutions are correct, complete, and sufficiently display appropriate, relevant concepts from the curriculum of MATA33.

1. Use the Lagrange Multipliers (LM) technique to maximize the "production function" $f(x,y) = 20x + 25y - x^2 - 3y^2$ subject to the "budget" constraint $2x + 4y = \dfrac{152}{3}$.
   You may assume that (i) "input amounts" x and y are positive, rational numbers, (ii) all of the budget is used, (iii) the production function has a unique maximum value subject to the budget and LM will find where this occurs.

   [15 points]

2. (a) Let $I = \int_0^1 \int_{x^2}^{\sqrt{x}} (x+y)\, dy dx$. Sketch the region of integration for I and evaluate I.

[10 points]

(b) Find the value of $\int\int_{\mathcal{R}} e^{y^2}\, dA$ where $\mathcal{R}$ is the triangular region with vertices $(0,0)$, $(0,2)$, and $(2,2)$. (Hints: draw $\mathcal{R}$ and choose the order of integration carefully.)

[7 points]

3. In economics, a general Cobb-Douglas production function has the form $P = cx^a y^b$ where $x > 0$ is "input money" and $y > 0$ is "labour quantity" and a, b, and c are positive constants with $a + b = 1$.

   (a) Verify that $xP_x + yP_y = P$ [6 points]

   (b) Determine whether the marginal production with respect to input money is increasing or decreasing as a function of labour quantity. Justify your answer with an appropriate partial derivative. [4 points]

4. Let $u(r, s) = f(r^2 - s^2, s^2 - r^2)$ where f is a "MATA33 function" for which the chain rule is valid. Show that $s\dfrac{\partial u}{\partial r} + r\dfrac{\partial u}{\partial s} = 0$. [7 points]

5. Find and classify all critical points of the function $w = f(x, y, z) = 3x^2 + y^2 + z^3 + zy^2 - 6z^2$. Show all calculations and sufficient details.

[15 points]

6. The parts of this question are independent of each other.

(a) Let t, α, and β be constants and x and y variables. Use the appropriate determinant to show that the system of linear equations

$$(t-2)x + 5y = \alpha$$
$$x - (t+2)y = \beta$$

has a unique solution for all real t and then use Cramer's rule to solve for x and y.

[9 points]

(b) Assume $P = \begin{pmatrix} 2 & 1 \\ 1 & -5 \end{pmatrix}$ and $Q = \begin{pmatrix} x & y \\ y & z \end{pmatrix}$ commute. Find integers a, b, and c with $a > 0$ such that $ax + by + cz = 0$

[7 points]

7. In all of this question let $z = f(x,y) = \dfrac{8y - 8x^2 + 32}{5 - y}$.

(a) Show that f does not have any critical points. [8 points]

(b) Use part (a) to verify the equality of mixed partials theorem. [5 points]

(c) Find the function $y = h(x)$ that represents the level curve of f passing through $(2,1)$. [6 points]

8. Three investors I_1, I_2, and I_3 each have a stock portfolio consisting of the same four stocks S_1, S_2, S_3, and S_4. Let a "quantity matrix" be $Q = \left[q_{ij}\right] = \begin{bmatrix} 18 & 26 & 4 & 1 \\ 5 & 2 & 9 & 53 \\ 10 & 17 & 26 & 33 \end{bmatrix}$ where

q_{ij} = the number of shares of stock S_j held by investor I_i throughout all of 2014.

(a) Assume the value in \$CAD of S_j at the end of the n^{th} day of 2014 is $w_j(n)$. Let $W(n)$ be the column matrix with entries $w_j(n)$. Find and interpret the meaning of $QW(n)$.

[7 points]

(b) On August 13, 2014 (the 225^{th} day of 2014) the American/Canadian dollar exchange rate is 1 \$US = 1.09 \$CAD. State the matrix A so that the entries in the product $AQW(225)$ represent the average value of each investor's stock portfolio on August 13, 2014 in \$US.

[4 points]

(c) Suppose that R represents a revised quantity matrix for all of 2015 where, compared to 2014, each investor has 2 more units of S_1, 10% less of S_2, the same amount of S_3, and 5% more of S_4. If $P + Q = R$, find P.

[5 points]

(This page is intentionally left blank)