Sorry...No Solutions are Provided

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

Final Examination

MATA33 - Calculus for Management II

Examiner: R. Grinnell

Date: August 22, 2012
Time: 2:00 pm
Duration: 180 minutes

Provide the following information:

(Print) Surname: ______________________________

(Print) Given Name(s): ______________________________

Student Number: ______________________________

Signature: ______________________________

Carefully read these instructions:

1. This exam has 15 numbered pages. It is your responsibility to check at the beginning of the exam that all of these pages are included.
2. Answer all questions in the work space provided. If you need extra space, use the back of a page or the last page, and indicate clearly the location of your continuing work.
3. Full points are awarded for Part B solutions only if they are correct, complete, and sufficiently display concepts and methods of MATA33.
4. You may use **one** standard hand-held calculator (graphing facility is permitted), but it **cannot** be able to perform any kind of matrix manipulations, differentiation, or integration. The following are forbidden at your exam writing space: laptop computers, Blackberrys, cell-phones, I-Pods, MP-3 players, extra paper, textbooks, or notes.
5. You may write in pencil, pen, or other ink.

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Print letters for Multiple Choice & True/False questions in these boxes.

1	2	3	4	5	6	7	8	9	10	11	12	13

Do not write anything in the boxes below.

Part A
42

Part B

1	2	3	4	5	6	7	8	9
13	16	13	14	11	16	10	9	6

Total
150

Part A - Multiple Choice Questions For the following 8 Multiple Choice questions, **clearly print the letter of the answer you think is most correct in the box at the top of page 2**. Each right answer earns 4 points. No answer or wrong answers earn 0 points. Justification is neither required nor rewarded, but a small workspace is provided for your calculations.

1. If $A = \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$ and $C = [C_{ij}] = 4A^{-1}B$ then $C_{11} + C_{22}$ equals

   (A) 4 (B) -6 (C) 6 (D) -12 (E) none of (A) - (D)

2. If $z = \dfrac{xy^2}{x+3}$ then the value of $z_{xy}(-1,6)$ is

   (A) 9 (B) 3 (C) 2.25 (D) -9 (E) none of (A) - (D)

3. If the equation $ux^3 = ye^u$ defines u implicitly as a function of two independent variables x and y, then $u_y(x,y)$ equals

   (A) $e^u x^{-3}$ (B) $\left(e^u + ye^u\right)x^{-3}$ (C) $e^u\left(x^3 - ye^u\right)^{-1}$ (D) none of (A) - (C)

4. Products **X** and **Y** have unit prices x and y, respectively. Assume their respective demand functions are $f(x,y) = e^{-2x-3y}$ and $g(x,y) = -y + 5x^{-2}$. We may conclude that **X** and **Y** are

   (A) competitive (B) complementary (C) neither competitive nor complementary

5. At the point $(2,1)$ the function $f(x,y) = x^2 - 4x + y^2 + 2y$ has

   (A) a local minimum (B) a local maximum (C) a saddle point (D) none of (A) - (C)

6. Let (a,b,c) be a critical point of a MATA33 function $w = f(x,y,z)$ where $a, b < -1$. Assume the Hessian matrix is $A = Hf(a,b,c) = \begin{bmatrix} ab & b & 0 \\ a & ab & 0 \\ 0 & 0 & a \end{bmatrix}$ At the point (a,b,c) we may conclude that

   (A) f has a relative minimum (B) f has a relative maximum

   (C) f has no relative extrema (D) the second-derivative test gives no information

7. The value of $\int_0^3 \int_1^3 (x + y^2)\, dy\, dx$ is

(A) 87 (B) 26/3 (C) 35 (D) 38/3 (E) 36 (F) none of (A) - (E)

8. The domain of the function $z = f(x, y) = \frac{1}{y} + ln(y^2 - x)$ is all points (x, y) that are

(A) strictly to the left of the curve $x = y^2$
(B) strictly to the right of the curve $x = y^2$ and not on the x-axis
(C) strictly to the left of the curve $x = y^2$ and not on the x-axis
(D) strictly to the left of the curve $x = y^2$ and not on the y-axis

Part A Continued - True/False Questions For the following 5 True/False questions, answer **T** (for True) if and only if the given assertion is always true. Otherwise answer **F** (for False). **Clearly print your letters in the answer boxes at the top of page 2**. Correct answers earn 2 points and incorrect/blank answers earn 0 points.

9. If $w = p(x, y, z)$ is a polynomial in the variables x, y, and z, then for the mixed triple partial derivatives we have that $p_{xyz} = p_{xzy}$

10. If $z = f(x, y)$ has a constrained critical point (a, b) subject to the constraint $g(x, y) = 0$ then $f_x(a, b) = f_y(a, b) = 0$.

11. Every system of two linear equations in three variables has infinitely many solutions.

12. If A and B are 3×3 matrices such that their product AB is invertible, then both A and B must be invertible too.

13. Let $0 < a < b$ and $0 < c < d$. The double integral $\int_a^b \int_c^d f(x, y)\, dy\, dx$ represents the volume of the solid region whose base is the rectangle given by a, b, c and d that lies beneath the surface $z = f(x, y)$.

(Be sure you have printed the Multiple Choice and True/False answers in the boxes on page 2)

Part B - Full Solution Questions

1. Examine the function $z = f(x, y) = \dfrac{x^3}{3} + y^2 - 2x - 6y - 2xy$ for local extrema.

[13 points]

2. (a) If $z = f(x,y) = e^{x^2 y}$ and $x = 2r + s$ and $y = \dfrac{r}{s}$, find $\dfrac{\partial z}{\partial r}$ and $\dfrac{\partial z}{\partial s}$ evaluated at $r = -1$ and $s = 1$. [9 points]

(b) Let $G(x,y) = \dfrac{xy}{\alpha x + \beta y}$ where α and β are positive constants and x and y are positive real variables.

Verify that $G_x(x,x) + G_y(y,y) = \dfrac{1}{\alpha + \beta}$ is a constant. [7 points]

3. Use the method of Lagrange Multipliers to maximize the function $w = f(x, y, z) = xy + yz$ subject to the two constraints $x+2y = 6$ and $x = 3z$. Your answer should show the point(s) (x, y, z) that maximize f and the maximum value of f. Show all work.

[13 points]

4. A division of KidzFun Toy Company produces toy planes and toy boats. The planes sell for \$10 each and the boats sell for \$8 each. It costs \$3 in raw materials to make a plane and \$2 in raw materials to make a boat. A plane requires 3 minutes to make and 1 minute to finish, and a boat requires 1 minute to make and 2 minutes to finish. KidzFun management has determined that at most 35 planes can be made per day. Furthermore, the company cannot spend more than 160 minutes per day finishing toy planes and boats (together, in total), and cannot spend more than 120 minutes per day making toy planes and boats (together, in total).

   Set-up the Linear Programming Problem (LPP) of determining the number of each toy made per day so as to maximize the total daily profit subject to all of the conditions above (i.e. state the objective function and all constraints). Solve the LPP of of determining the number of each toy made per day so as to maximize the total daily profit. Show an accurate, labeled feasible region, all details of calculations, the maximum daily profit, and the corresponding number of toys made per day.

[14 points]

5. For the function $f(x, y, z) = x^2 - x - xy + y^2 - y + z^4 - 4z$ find all critical points and classify them. Show all work. [11 points]

6. In all of this question let (*) represent the equation $x^2z + yz^2 = 14$ and assume z is defined implicitly as a function of independent variables x and y.

(a) Verify that the point $(-1, 3, 2)$ satisfies (*) and show that $z_x = \dfrac{\partial z}{\partial x} = \dfrac{-2xz}{x^2 + 2yz}$

[7 points]

(b) Assume that $z_y(-1, 3, 2) = \dfrac{-4}{13}$. Calculate the value of the constant K such that $z_{xy}(-1, 3, 2) = \dfrac{K}{(13)^3}$

[9 points]

7. Suppose a person will buy $x > 0$ units of product **X** and $y > 0$ units of product **Y**. Both x and y can be fractions of units. A "budget constraint" is given by $ax + by = c$ where a, b, and c are positive constants. A "utility function" is given as $U(x, y) = xy + 2x$. This measures the person's total satisfaction from their purchase of **X** and **Y**. Economic theory shows that U can be maximized subject to the budget.

   Use Lagrange Multipliers to show that U is maximized subject to the budget when $x = \dfrac{c+2b}{2a}$ and $y = \dfrac{c-2b}{2b}$. What inequality must c and b satisfy to guarantee that x and y are positive? Show all work.

   [10 points]

8. Determine the values of the parameter $t \in \mathbf{R}$ for which the system of equations

$$\begin{aligned} 2tx + y &= 1 \\ 3tx + 6ty &= 2 \end{aligned}$$

has a unique solution (and justify your findings). Then use Cramer's rule to describe the unique solution. [9 points]

9. Let $f(x,y)$ be a MATA33 function and let $I = \int_0^2 \int_{y^2}^{2y} f(x,y)\, dx dy$. Sketch and label the region over which integration in I takes place and then re-write I with the order of integration reversed.

[6 points]

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