University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA33 - Calculus for Management II

Examiner: R. Grinnell

Date: August 19, 2009
Duration: 3 hours

Provide the following information:

Lastname (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number (PRINT): ______________________________

Signature: ______________________________

Read these instructions:

1. This examination paper has 13 pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.
2. Put your solutions and/or rough work in the answer space provided. If you need extra space, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of any continuing work.
3. You may use **one** standard hand-held calculator. All other electronic devices, extra paper, notes, and textbooks are forbidden at your workspace (either visibly or in any sort of carrying case or by accident).

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	8	**TOTAL**
45	11	13	12	13	13	16	18	9	150

Part A: Multiple Choice Questions For each of the following **print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 5 points and no answer/wrong answers earn 0 points. Justification is neither required nor rewarded, but a small workspace is provided for your calculations.

1. If $z = (4x - 3y)^2$ and $x = rs^2$ and $y = r - 2s$ then the value of $\dfrac{\partial z}{\partial r}$ at $r = 0$ and $s = 1$ is

   (a) 12 (b) 20 (c) 24 (d) -12 (e) none of (a) - (d)

2. Which of the following are critical points of $f(x, y) = x^3 - 3xy - y^3$?

   (a) $(0, 0)$ (b) $(1, -1)$ (c) $(-1, 1)$ (d) $(1, 1)$

   (e) (a) and (b) (f) (a) and (c) (g) (a) and (d)

3. Let $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -5 \\ 3 & k^2 \end{pmatrix}$. What values of the variable k, if any, will make the matrices A and B commute ?

   (a) ± 1 (b) ± 5 (c) $\pm\sqrt{3}$ (d) $\pm\sqrt{5}$ (e) none of (a) - (d)

4. If $z = (xy)^{1/2} e^{xy}$ then $\dfrac{\partial z}{\partial y}$ equals

(a) $\dfrac{z}{2} + xz$ (b) $\dfrac{yz}{2} + xz$ (c) $\dfrac{z}{2y} + xz$ (d) $\dfrac{z}{2y} + z$ (e) none of (a) - (d)

5. Assume the equation $w^2 = 12x - 15y$ defines w implicitly as a function of independent variables x and y. The value of $w_{xx}(2, 1, -3)$ is

(a) $-4/3$ (b) $4/3$ (c) $-3/4$ (d) $3/4$ (e) none of (a) - (d)

6. Let R represent the feasible region defined by the inequalities $1 \le y \le 3$ and $y \le 2x + 1$ and let $Z = -ax + \dfrac{a}{2}y$ where $a > 0$ is a constant. We may conclude that

(a) Z is maximized and minimized on R.

(b) Z is maximized at a unique point in R, but Z is not minimized on R.

(c) Z is maximized at infinitely many points in R, but not minimized on R.

(d) Z is minimized at infinitely many points in R, but not maximized on R.

(e) Z is not optimized on R.

7. The critical point(s) (x, y) of $f(x, y) = x - 3y - 1$ subject to the constraint $x^2 + 3y^2 = 16$ must satisfy

(a) $y = x$ (b) $y = 2x$ (c) $y = -\frac{1}{2}x$ (d) $y = -x$

(e) an equation that is not given in any of (a) - (d)

8. Assume A is an $n \times n$ matrix where $n \geq 2$. Exactly how many of the following mathematical statements are always true ?

(i) $det(2A) = 2n det(A)$

(ii) $det(I_n + A) = 1 + det(A)$

(iii) If $AX = 0$ has infinitely many solutions, then A is not invertible.

(iv) If $AB = AC$, then $B = C$

(v) If R is the reduced form of A and R has exactly n non-zero rows, then A is invertible.

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

9. The value of $\int_1^e \int_0^2 y ln(x)\, dy dx$ is

(a) 3 (b) 2 (c) $\frac{2}{e} - 2$ (d) $e + 1$ (e) $\frac{3e+1}{2}$ (f) none of (a) - (e)

Part B: Full-Solution Questions Write clear and neat solutions in the answer spaces provided. Show all of your work. Full points are awarded only for solutions that are correct, complete, and sufficiently display relevant concepts from MATA33.

1. Let $f(x, y) = x^2 + 2y^2 - x^2y$ Find the critical point(s) of f. For each one, use the 2^{nd}−derivative test to determine whether it corresponds to a relative maximum, minimum, or a saddle point. [11 points]

2. Find the maximum and minimum values of the function $Z = -4x - 6y$ subject to the constraints: $y \geq 1$, $1 \leq x \leq 6$, $y \leq 2x + 1$, and $2x + 3y \leq 27$

(To earn full points, your solution must include a neat, labeled diagram of the feasible region, the location of all points where Z is optimized, all calculations, and appropriate justifications.)

[13 points]

3. (a) Let k be an arbitrary real constant and let x and y be variables. Find all solutions to the system [6 points]

$$\begin{aligned} kx + 2y &= x \\ x - ky &= y \end{aligned}$$

(b) What equation must real constants a, b, and c satisfy in order that the system of linear equations

$$\begin{aligned} x + 3y + z &= a \\ 2x - y - z &= b \\ 4x + 5y + z &= c \end{aligned}$$

has infinitely many solutions ? [6 points]

4. A cylindrical can has a volume of 3 litres ($= 3,000\ cm^3$). The material for the base costs 0.2 cents per cm^2. The material for the lid and the wall of the cylinder costs 0.1 cents per cm^2. **Use the method of Lagrange multipliers** to calculate the least total material cost subject to the given volume constraint. Round your final answer up to the nearest cent. You may assume the critical point obtained by the Lagrange multiplier technique actually does correspond to the least cost. [13 points]

   (These formulas may be useful: Area of a circle $= \pi r^2$ Area of cylinder wall $= 2\pi rh$)

5. In all of this question let R be the triangular region in the plane having vertices $(-1,-2)$, $(0,0)$, and $(0,-2)$.

   (a) Draw a small picture of R (off to the right would be good) and give the set-notation that describes R by: [5 points]

      (i) Vertical sections

      (ii) Horizontal sections

   (b) Evaluate $I = \int\int_R e^{y^2}\, dA$ [8 points]

6. In all of this question let $f(x,y) = xy + \dfrac{a^3}{x} + \dfrac{b^3}{y}$ where a and b are arbitrary non-zero real constants of opposite signs.

(a) Find the unique critical point of f. [6 points]

(b) Use your answer to part (a) and the 2^{nd}−derivative test to verify that f has a local maximum value of $3ab$ [8 points]

(c) Show (mathematically) why f does not have an absolute maximum. [2 points]

7. In all of this question let $u = f(x,y) = \sqrt{x^2 - 2y + 2}$

(a) Find the level curve of f that passes through the point $(8, -1)$ and write it in the form $y = g(x)$. [5 points]

(b) Verify that $u(u_x + u_y) = x - 1$ [6 points]

(c) Let S represent the domain of the function $h(x,y) = f(x,y) + \sqrt{x} + \sqrt{y}$. Describe S in set-notation using appropriate inequalities. Give a sketch that clearly shows S. (light shading would be good). [7 points]

8. In all of this question let P be an arbitrary $n \times n$ matrix where $n \geq 2$.

(a) Assume P is invertible and prove mathematically that the matrix equation $PX = Q$ has a solution for every $n \times 1$ matrix Q. [3 points]

(b) Assume the matrix equation $PX = B$ has a solution for each $n \times 1$ matrix B and prove mathematically that P is invertible. (Note: parts (a) and (b) are independent of each other – they are not related.) [6 points]

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