**University of Toronto at Scarborough**
**Department of Computer & Mathematical Sciences**

**MATA33S** **Assignment 9 (2 Pages)** **Winter 2018**

**Study:** Section 17.6 and the home page pdf document entitled "Hessian Optimization". Read ahead in Sections 17.7 and 17.9. We omit Section 17.8. Also see the **Notes** below.

**Problems:** (Many and challenging $\heartsuit$)

1. Section 17.6, Pages 775 - 777 # 1 - 3, 6 - 8, 10 - 14, 16, 18 - 20, 22, 23, 24, 28, 29, 33, 34, 36.

2. A delivery company accepts only rectangular boxes who length plus "girth" do not sum over 108 cm (The "girth" of a rectangular box is the perimeter of a cross-section). Find the dimensions of an acceptable box of largest volume.

3. In this question let $f(x,y) = x^2 - y^2 - 2x + 4y + 6$

   (a) Use the critical point concepts and the second derivative test to find out that $f$ has a critical point at $(1,2)$ but no relative extrema there.

   (b) Prove algebraically (i.e. not using calculus) that $f$ has no relative extrema at $(1,2)$.

4. (a) Show that the critical point analysis and second derivative test provide no information about extrema of the function $f(x,y) = x^4 + y^4$

   (b) Use algebra (and no calculus) to find the local extrema of $f$ in part (a). Prove also that the local extrema you find in part (a) is actually absolute extrema.

5. (a) Repeat part (a) in Problem 4 for the function $f(x,y) = x^4 - y^4$

   (b) Use algebra (and no calculus) to show that $f$ has no local extrema at the point $(0,0)$.

6. In this question let $f(x,y) = x^2 - e^{(y^2-1)}$

   (a) Find all of the critical points of the function $f$

   (b) Find all of the critical points of the function $g(x) = f(x,x)$

   (c) What is surprising in your result for (b) compared to that of (a)?

7. A rectangular box with no top is constructed from exactly $12m^2$ of material (i.e. there is no waste).

   (a) With the length, width, and height represented by positive numbers $x$, $y$, and $z$ respectively, show that the volume, $V$, of the box subject to the material constraint above is given by $V = \dfrac{xy(12-xy)}{2x+2y}$.

   (b) Verify that if $V_x(x,y) = V_y(x,y) = 0$ then $x = y$.

(c) Re-read the paragraph entitled, "Applications" on page 773 and convince yourself that there is a maximum volume of the box. Under this assumption, verify that the maximum volume is $4m^3$. (Note: the idea here is to not use the second-derivative test. That test would be quite complicated because of the second derivatives)

(d) Use (b) to write $V$ as a function of $x$ only and then use optimization methods from MATA32 to prove that the maximum value of the volume is $4m^3$.

8. Page 795, # 25, 26

9. For each of the following functions of three variables, find the critical point(s). Then for each critical point, use the second derivative test to determine whether it yields a local (i.e. relative) maximum, minimum, or neither.

   (a) $f(x, y, z) = x^3 + xy^2 + x^2 + y^2 + 3z^2$

   (b) $f(x, y, z) = x^3 + xz^2 - 3x^2 + y^2 + 2z^2$

   (c) $f(x, y, z) = x^2y + y^2z + z^2 - 2x$

   (d) $f(x, y, z) = xy - xz$

**Notes:**

1. Quiz 5 is in Week 11 (Friday March 23 - Thursday March 29). It will not have any problems/questions from this Assignment 9. See Assignment 8 for further details as to the material of responsibility for Quiz 5.

2. There will be two more posted assignmnents/solutions after this one: Assignments 10 and 11.

3. Midterm test Solutions and some statistics will be posted at our home page in the week of March 19 - 23.

4. Please consider doing the Sociology survey. A link to that survey was emailed to students on Monday March 12, 2018 around 8:15pm. A 1% bonus will be added to your final grade upon completion of the survey.