University of Toronto at Scarborough Department of Computer & Mathematical Sciences

MATA33S

Assignment 6 (Regular) (2 pages)

Winter 2018

There are two Assignments 6 versions: this one called "Regular" and another called "Enhanced". They are very similar and you should work hard on both of them. Please also read the **Notes** below.

Terminology and Concepts to Learn: (Section 2.8): ordered 3-tuple and ordered n-tuple, function of several variables and its graph, 3-dimensional rectangular coordinate system, planes in 3-dimensions, intercepts and traces, level curves, graph of a function of 2 variables (Section 17.1): definition of partial derivatives, finding partial derivatives, partial derivative notation and evaluation.

Study: Sections 2.8 and 17.1 for this assignment (This is the beginning of our study of Calculus of Several Variables, which we will continue for the remainder of the course). Read ahead in Sections 17.2 and 17.3.

Problems:

- 1. Section 2.8 # 1 5, 8 12, 15 21, 23 26, 28, Sketch the surface $3x^2 + 2y^2 = 1$.
- 2. Section 17.1 # 1, 2, 5, 6, 8 10, 13 17, 20, 21, 27 31, 35, 36, 38, 34, 37.
- 3. For this exercise, reference the Amount of an Annuity formula on Page 224, Equation (3) in the bottom blue box. Find the first partial derivatives of S with respect to R, r, and n.

In Problems 4, 5, and 6 below we investigate further the concept of a level curve (Recall that this concept was used extensively in our study of linear programming. It would be a good idea to re-read pages 300-301.) Given a function of two variables z = f(x, y) and a real number c, the **level curve with constant c** is the set $L(c) = \{(x, y) \mid f(x, y) = c\}$. Thus, L(c) is the set of points (x, y) in the x, y-plane for which f(x, y) has value c.

- 4. Let $z = f(x, y) = x^2 y + 1$.
 - (a) Find and sketch the level curves L(0), L(-2), and L(5).
 - (b) Find the functions g(x, y) = f(f(x, y), y) and h(x, y) = f(x, f(x, y)).
 - (c) If f(x,y) = f(y,x) and $x \neq y$, show that y = -x 1.
 - (d) Suppose f(x, y) represents the profit when x and y numbers of units of two products, **X** and **Y** (respectively), are sold. Find and sketch the set $\mathbf{P} = \{(x, y) \mid \text{profit is positive}\}$. Find and sketch the set of points $(x, y) \in \mathbf{P}$ such that $f_x(x, y) = f_y(x, y) = 0$.
- 5. Let $z = u(x, y) = \sqrt{2x y + 1}$.
 - (a) Find and sketch the domain of u(x, y).
 - (b) Find and sketch the level curves L(0) and L(1) on the same set of axis. On another set of axis, sketch L(c) for all $c \in [0, 1]$. What can you say about the level curves L(c) if c < 0?
 - (c) Find $u_x(x,y)$, $u_y(x,y)$, their domains, and verify that $(u_x(x,y) + u_y(x,y))u(x,y)$ is a constant.

- 6. Let $z = F(s,t) = \sqrt{s^2 + t^2}$.
 - (a) Find and sketch the level curves L(0) and L(1) on the same set of axis. On another set of axis, sketch L(c) for all $c \in [1, 2]$.
 - (b) Find z_s , z_t , and verify that $sz_s + tz_t = z$.
- 7. Find the first partial derivatives for the functions: (a) $z = x^y$ and (b) $z = log_x(y)$.
- 8. Let $E = \left(1 + \frac{r}{n}\right)^n 1$ be the effective rate function where r is the APR and n is the number of times interest compounds per year (i.e. the compounding frequency). See page 211 at the top.
 - (a) Verify that $(n+r)E_r = n(E+1)$ where E_r denotes the partial derivative of E with respect to r.
 - (b) Calculate E_n (the partial derivative of E with respect to n) for an APR of 12% and n = 12. Round your final answer to five decimals.

Notes:

- 1. Quiz 4 is in Week 9 (Friday March 9 Thursday March 15). You should expect questions from material in both versions of Assignment 6 and possibly from Assignment 7.
- 2. Assignment 7 will be posted not later than Monday March 5.