# University of Toronto at Scarborough Department of Computer \& Mathematical Sciences <br> Assignment 4 (2 pages) 

MATA33S
Winter 2018

Work on the course material and problems below. Read the Notes below pertaining to Quiz 2 and our Midterm Test.

Study: Sections 6.6 on matrix inverse. Review sections 6.4 and 6.5 as necessary. Read ahead on Determinants at Wikipedia or online at any top Google hits for "determinant".

Terminology and Concepts to Learn: invertible matrix, using matrix inverse to solve a matrix equation, determining whether a matrix is invertible (and if so, how to find the matrix inverse), properties of matrix inverse.

## Problems:

1. Section $6.6 \# 1,2,5-8,13,18-22,27,28,31,32,35,36,39-42$.
2. Page 292, \# 32 .
3. Find the solution to the system of linear equations whose augmented matrix is

$$
A=\left[\begin{array}{rrrrrr}
1 & 6 & 2 & -5 & -2 & -4 \\
0 & 0 & 2 & -8 & -1 & 3 \\
0 & 0 & 0 & 0 & 1 & 7
\end{array}\right]
$$

4. Does the the system of linear equations:

$$
\begin{gathered}
x+3 y+4 z=a \\
-4 x+2 y-6 z=b \\
-3 x-2 y-7 z=c
\end{gathered}
$$

have a solution for all possible values of $\mathrm{a}, \mathrm{b}$ and c ? Discuss.
5. If $A$ and $B$ are $2 \times 2$ invertible matrices, is $A+B$ also invertible? If so, try to prove it. If not, produce a counter example.
6. Suppose $A$ is a $3 \times 3$ matrix such that $A^{4}-2 A^{3}+5 A^{2}-2 I=0$. Show that $A$ is invertible. (Hint: think carefully about the definition of invertibility of a matrix on page 278).
7. Let $A=\left[\begin{array}{ll}4 & -3 \\ 2 & -1\end{array}\right], P=\left[\begin{array}{ll}3 & 1 \\ 2 & 1\end{array}\right]$, and let $D=\left[\begin{array}{ll}2 & 0 \\ 0 & 1\end{array}\right]$.
(a) Verify that $A=P D P^{-1}$
(b) Use part(a) to calculate $A^{3}$ and $A^{10}$ (the point of the question is not to multiply $A$ by itself many times).
(c) Find an expression for $A^{n}$ where $n$ is an arbitrary natural number (Hint: use part(a) and your results in part(b)).

## Notes:

1. Quiz 2 is in Week 5 (Friday February 2 - Thursday February 8). There are no Quiz 2 questions on matrix inverse (Section 6.6) or any questions from this Assignment 4. See Assignment 3 for Quiz 2 details.
2. Recall that you can only write a quiz in the tutorial you are officially registered in. If you write a quiz in a tutorial you are not officially registered in, your score will be 0 .
3. You cannot use a calculator or other electronic device, or have scrap paper or textbooks or notes during the writing of any quiz, the midterm test, or final exam.
4. The Midterm Test is on Monday February 26, 5:00pm - $6: 50 \mathrm{pm}$. A midterm test information sheet will be posted at our home page and emailed in due course. You should expect that the test will cover (at least) all lecture material and text material up to and including Week 6 (Friday February 9 - Thursday February 15) and all assignments that are relevant to that material.
