

University of Toronto at Scarborough
Department of Computer & Mathematical Sciences

MATA33S

Assignment 3 (2 Pages)

Winter 2018

Work on the material and problems below.

Quiz 2 is in Week 5 (Friday Feb 2 - Thursday Feb 8). It will cover matrices from Assignment 2 (Problems 1 - 7 only), all of this Assignment 3, related lecture material on matrices, and maybe some ideas/problems from Assignment 4.

Terminology and Concepts to Learn: system of linear equations, coefficient and augmented matrix, elementary row operations, reduced matrix, how to reduce a matrix, solving a system by reduction, "nature of solutions" (i.e. none exists, unique solution, infinitely many solutions), basic and free variables, parameterized family of solutions, homogenous system, trivial and non-trivial solutions, "number" of solutions of homogenous system.

Study: Review Sections 6.1 - 6.3 as necessary and currently study Sections 6.4 and 6.5. Study ahead of the lectures in Section 6.6. Study as the matrix topic of "determinants" on-line. There will also be lecture notes on that topic.

Problems:

1. Section 6.4 # 1 - 18, 20 - 22, 27, 28, 30, 33.

2. Section 6.5 # 1, 2, 6, 7, 9 - 12, 15 - 20.

3. Consider the augmented matrix $A = \begin{bmatrix} 1 & -4 & 7 & a \\ 0 & 3 & -5 & b \\ -2 & 5 & -9 & c \end{bmatrix}$ Find an equation involving the variables a , b , and c so that A corresponds to a system of three linear equations in three unknowns having infinitely many solutions.

4. Determine the value(s) of real numbers a and b so that the system of equations:

$$x + 3y = a$$

$$4x + by = 8$$

has (i) no solution; (ii) a unique solution; (iii) infinitely many solutions.

5. Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -6 \\ 2 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & -7 \\ 1 & 2 \end{bmatrix}$ and let $C = AB$. Show how each column of C can be written as a sum of scalar multiples of the columns of A . We say that each column of C is a **linear combination** of the columns of A .

6. Investors I_1 and I_2 each have a portfolio consisting of the same four stocks: S_1, S_2, S_3 , and S_4 . Consider the matrix $Q = [q_{i,j}] = \begin{bmatrix} 200 & 300 & 100 & 200 \\ 100 & 200 & 400 & 0 \end{bmatrix}$ where $q_{i,j}$ is the number of shares of stock S_j held by investor I_i .

- (a) Suppose stock S_j is valued at $v_j(n)$ dollars per share at the close of the TSX (i.e. Toronto Stock Exchange) on the n^{th} day of 2016. Let $V(n) = [v_j(n)]$. Find and interpret the meaning of the matrix product $QV(n)$.
- (b) Let $A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \end{bmatrix}$. Find and interpret the meanings of the products QA and BQ .

Notes:

1. Make a serious effort to study ahead of the current lectures! Many students have said over many years that this technique has proven to be very helpful in their learning MATA33S. Use the Syllabus and Schedule (posted at our home page) to work a section (or even two) beyond the current lecture material.
2. Supplement your studying and learning by going on-line! The Khan Academy is a good place to start. Look under Math and Algebra.
3. Take advantage of office hours (see the Office Hour schedule posted at our home page).
4. The MATA33S Midterm Test is on Monday February 26, 5:00pm - 6:50pm. A test information document will be circulated/posted in due course.