MATA32 Test 1 Solutions Oct 10, 2008

Part A - Multiple Choice For each of the following **carefully circle the letter** next to the answer you think is most correct. Each correct answer earns 4 points and no answer/incorrect answers earn 0 points. Justification is neither required nor rewarded, but a small workspace is provided for your calculations.

1. To the nearest cent, the present value of \$ 60 due in three years at 3.6 % APR compounding quarterly is

   (a) \$ 39.25 (b) \$ 57.88 (c) \$ 58.41 (d) \$ 53.88

$$PV = 60\,(1.009)^{-12} \approx 53.88$$

$$\left(\frac{.036}{4} = .009\right)$$

2. To three decimals, what interest rate compounded continuously is equivalent to 5.1 % APR compounded every three months?

   (a) 5.057 % (b) 6.743 % (c) 5.068 % (d) 3.801 %

$$\frac{.051}{4} = .01275 \qquad e^{r} = (1.01275)^4$$

$$\therefore\ r = 4\ln(1.01275) \approx 5.068\%$$

3. The future value of an annuity due is \$ 50 and it consists of 5 payments at 6 % APR compounding monthly. The monthly payment to the nearest cent is

   (a) \$ 9.23 (b) \$ 8.19 (c) \$ 9.85 (d) none of (a), (b) or (c).

$$50 = R\left[\frac{(1.005)^5 - 1}{.005}\right](1.005) \rightarrow R \approx 9.85$$

4. If $h(x) = \dfrac{x^2 - 9x + 20}{x^2 - 3x - 4}$ then the value of $\lim_{x \to 4} h(x)$ is

   (a) 4/5 (b) −4/5 (c) −1/5 (d) none of (a), (b), or (c)

$$h(x) = \frac{(x-4)(x-5)}{(x-4)(x+1)} \rightarrow \lim_{x \to 4} h(x) = \lim_{x \to 4} \frac{x-5}{x+1}$$

$$= -\frac{1}{5}$$

5. The least number of months it takes a principal of \$ P to increase by 40 % at 4.2 % APR compounding semi-annually is

(a) 102 (b) 97 (c) 96 (d) none of (a), (b) or (c)

$1.4\cancel{P} = \cancel{P}(1.021)^{2t}$, $t = \#$ of years $\because$ compounding is SEMI ANNUALLY,

$t = \dfrac{\ln(1.4)}{2\ln(1.021)} \approx 8.095$ we round to next HIGHEST 6 months

i.e. 8.5 years $\rightarrow$ 102 months.

6. A bank account gives 5.2 % APR compounding weekly. Assuming the account starts empty, the least full dollar amount your parents must deposit in the account now so that you can make a withdrawl of \$50 at the end of each week for the next 52 weeks in a year is

(a) \$ 2,668 (b) \$ 2,600 (c) \$ 2,485 (d) \$ 2,533

$$PV = 50\left[\frac{1-(1.001)^{-52}}{.001}\right] \approx 2,532.33$$

7. The limit $\lim_{x\to\infty}\left(\dfrac{x^3}{x^2-5x}-x\right)$

(a) equals 0 (b) equals ∞ (c) equals 5 (d) does not exist

$$\frac{x^3}{x^2-5x} - x = \frac{\cancel{x^3}-\cancel{x^3}+5x^2}{x^2-5x} \longrightarrow 5 \text{ as } x \to \infty$$

8. Given an effective rate r_e the APR $r\%$ compounded every other month that is equivalent to r_e is

(a) $r = 6\left(\sqrt[6]{1+r_e}+1\right)$ (b) $r = \sqrt[6]{1+r_e}$ (c) $r = 6\left(\sqrt[6]{1+r_e}-1\right)$ (d) $r = \sqrt[6]{r_e}-5$

$1 + r_e = \left(1+\frac{r}{6}\right)^6$ Solve for r:

$$\sqrt[6]{1+r_e} = 1 + \frac{r}{6} \longrightarrow r = 6\left(\sqrt[6]{1+r_e}-1\right)$$

Part B - Full Solution Problem Solving

1. A debt of \$ 8,000 due 4 years from now and \$ 4,000 due 8 years from now is to be repaid by three payments:

   (1) the first payment is at the end of 2 years from now;

   (2) the second payment (which is 3/4 of the first) is made at the end of 38 months from now;

   (3) the third payment (which is 2/3 of the second) is made at the end of 75 months from now.

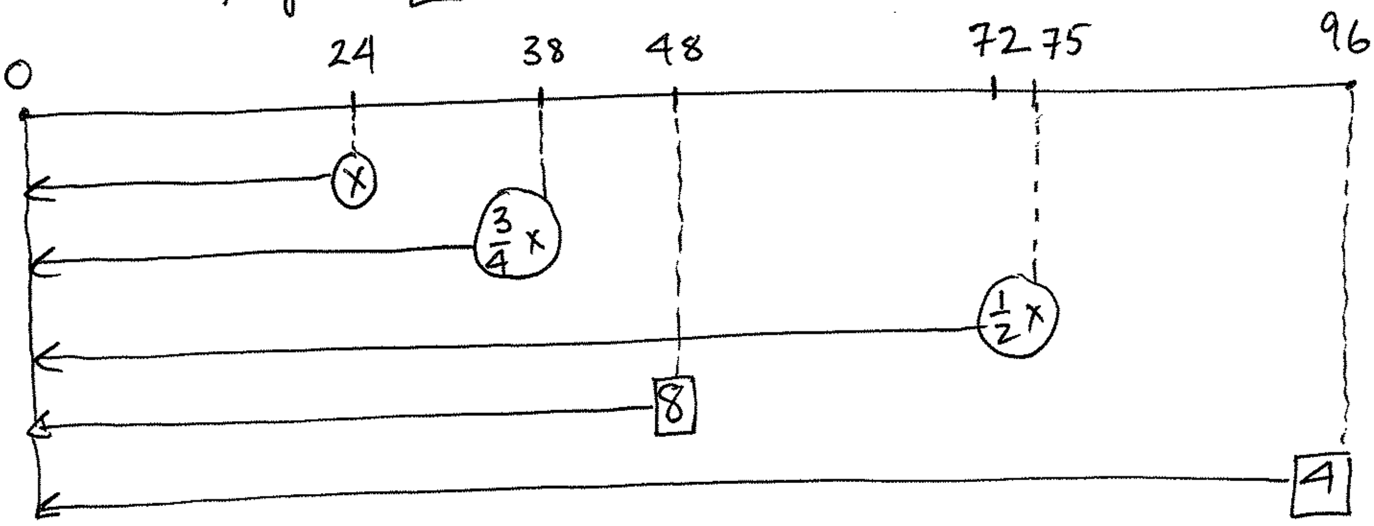
   If interest is 4.8 % APR compounding monthly, calculate the value of each payment.

   (Round your final answers up to the nearest dollar. A money-time diagram is required for full points). [12 points]

Let x = amount of 1st payment in \$ 1,000

Money-time diagram is calibrated to 0 and is in months

O → "pay" □ → "debt"



Equation of value: $\left(i = \frac{.048}{12} = .004\right)$

$$x(1.004)^{-24} + \left(\tfrac{3}{4}x\right)(1.004)^{-38} + \left(\tfrac{1}{2}x\right)(1.004)^{-75}$$
$$= 8(1.004)^{-48} + 4(1.004)^{-96}$$

We obtain $x \approx 4.850845$

$\therefore$ 1ST payment is about \$ 4,851
2nd " " " \$ 3,639
3rd " " " \$ 2,426

2. (a) Imagine winning a very large lottery. There are two banks in which to consider investing your winnings: Bank A pays 5.54 % APR compounding monthly and Bank B pays 5.52% APR compounding daily (365 days = 1 year). Which bank is the better choice to invest your lottery winnings and why?

[8 points]

The "better choice" is the bank with the higher effective rate.

$$(r_e)_A = \left(1 + \frac{.0554}{12}\right)^{12} - 1 \approx .056828572$$

$$(r_e)_B = \left(1 + \frac{.0552}{365}\right)^{365} - 1 \approx .056747535$$

Bank A is "better" because $(r_e)_A > (r_e)_B$ (as seen in the 4^{th} dp)

(b) Assume now there is a third bank to consider (Bank C) which offers r % compounded quarterly. What value of r makes Bank A and Bank C equally attractive for the investment of your lottery winnings? Round your final answer to three decimals.

[6 points]

Solve for r where

$$\left(1 + \frac{r}{4}\right)^4 = \left(1 + \frac{.0554}{12}\right)^{12}$$

$$r = \left[\left(1 + \frac{.0554}{12}\right)^3 - 1\right]4$$

$$\approx .055656157$$

$$\boxed{\therefore\ r = 5.566\%}$$

3. Find the annual continuously compounding interest rate that would cause a principal to increase by exactly 132% at the end of 4,234 days (365 days = 1 year). Exprss your answer as a percentage rounded to two decimals. [6 points]

$\frac{4234}{365} = 11.6$ years

Increase by 132%

So $P \to 2.32P$

$P =$ principal

Solve for r:

$$2.32\cancel{P} = \cancel{P}e^{11.6r}$$

$$r = \frac{\ln(2.32)}{11.6} \approx .07254886$$

$$\boxed{\therefore r = 7.25\%}$$

4. Find the value(s) of the constant c so that $\lim_{x\to 2} f(x)$ exists where

$$f(x) = \begin{cases} \frac{x^3-8}{x-2} & if \quad x < 2 \\ c^2x^2 & if \quad x > 2 \end{cases}$$

Justify your solution completely. [8 points]

$\because x \to 2$ and the definition of f changes functions at 2, we evaluate 1-sided limits and equate.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}\left(\frac{x^3-8}{x-2}\right) = \lim_{x\to 2^-}(x^2+2x+4)$$
$$= 12$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(c^2x^2) = 4c^2$$

Solve for c where $4c^2 = 12 \to \boxed{c = \pm\sqrt{3}}$

5. For each of the following, evaluate the limit or state why it does not exist.

(a) $\lim_{x\to 3} \frac{1-\sqrt{x-2}}{x-3}$ [8 points]

$$= \lim_{x\to 3}\left(\frac{1-\sqrt{x-2}}{x-3}\cdot\frac{1+\sqrt{x-2}}{1+\sqrt{x-2}}\right)$$

$$= \lim_{x\to 3} \frac{1-(x-2)}{(x-3)(1+\sqrt{x-2})}$$

$$= \lim_{x\to 3} \frac{-(x-3)}{(x-3)(1+\sqrt{x-2})}$$

$$= \lim_{x\to 3} \frac{-1}{1+\sqrt{x-2}}$$

$$= \boxed{-\frac{1}{2}}$$

(b) $\lim_{x\to 0^-} \left(\frac{1}{|x|}+\frac{1}{x}\right)$ $= \boxed{0}$ [5 points]

Here's why:

$x \to 0^-$ so $x \to 0$ and $x \approx 0$ and $x < 0$

$\therefore$ $|x| = -x$ so $\frac{1}{|x|}+\frac{1}{x} = -\frac{1}{x}+\frac{1}{x} = 0$

$\therefore$ $\forall\, x<0$ (and even $x\approx 0$), $\frac{1}{|x|}+\frac{1}{x}=0$

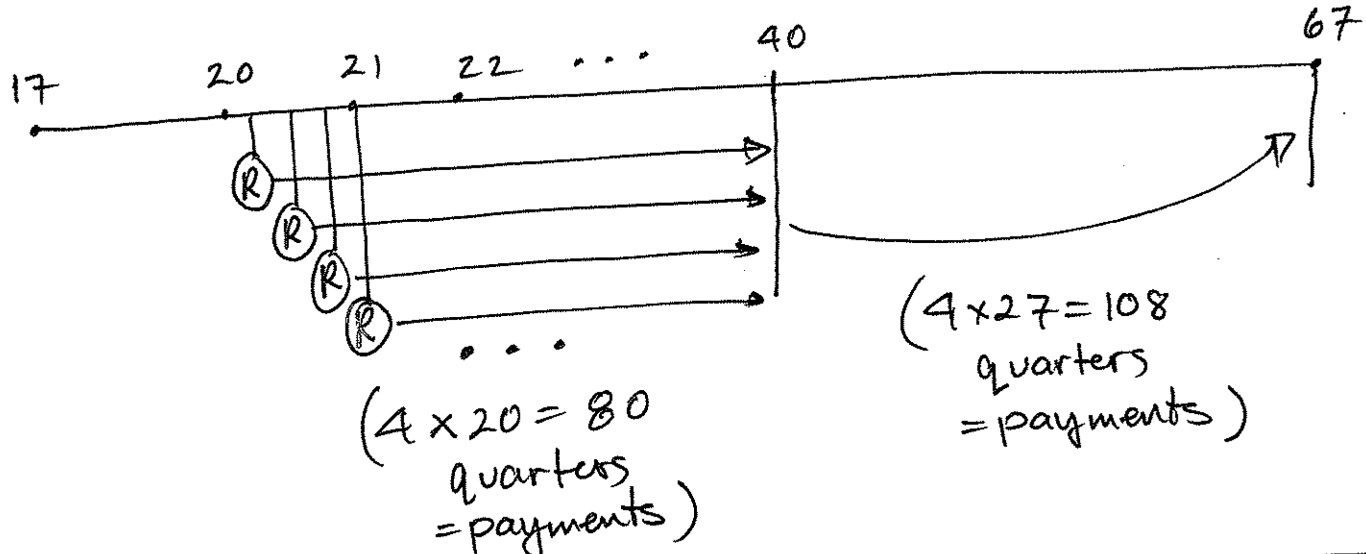
so we get $\boxed{0}$ above.

6. A bank account pays interest at 6% APR compounding quarterly. On your 17-th birthday you deposit \$ 1,000 into the empty account. Beginning with the first quarter after your 20-th birthday, you make a \$ 750 deposit into the account at the end of each quarter up to and including your 40-th birthday. Then starting with the first quarter after your 40-th birthday, you deposit at the end of each quarter \$ 1,500 up to and including your 62-nd birthday. There are no further deposits after your 62-nd. Calculate how much you will have in the bank account on your 67-th birthday. Round your final answer up to the nearest dollar. [12 points]

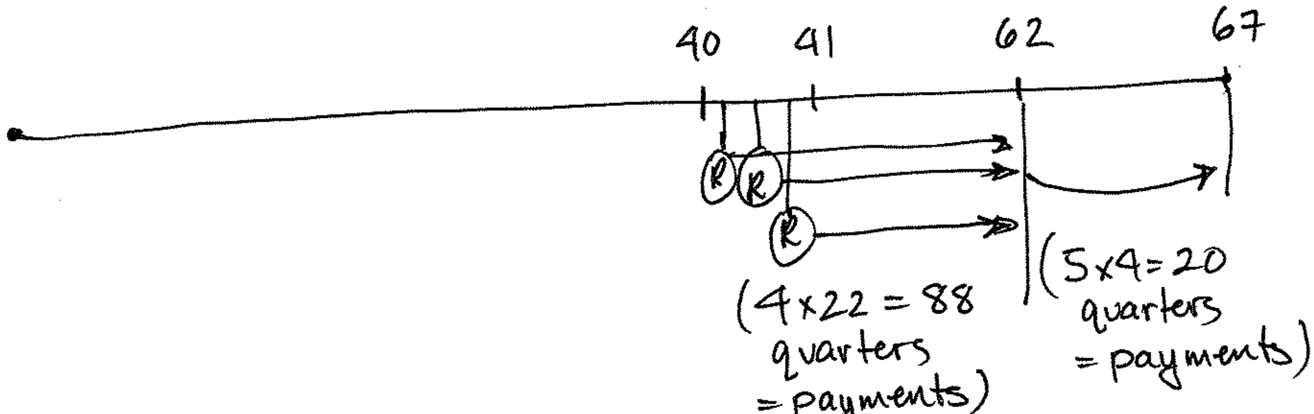
$\boxed{A}$ = Final amount = $A_1 + A_2 + A_3$ as follows.

$A_1 = 1000(1.015)^{200}$ 17 (50×4 = 200 quarters) 67

$$A_2 = 750\left[\frac{(1.015)^{80}-1}{.015}\right](1.015)^{108} \quad (R = 750)$$



$$A_3 = 1500\left[\frac{(1.015)^{88}-1}{.015}\right](1.015)^{20} \quad (R = 1500)$$



$\boxed{A} \approx 956{,}049.96$

$\boxed{\therefore\ A \approx \$956{,}050}$