University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MATA32F - Calculus for Management I - Midterm Test

Examiners: N. Cheng
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Date: October 19, 2016
Duration: 110 minutes
Time: 5:00 pm

Last Name (PRINT BIG) _____

First Name(s) (PRINT BIG) _____

Student Number _____

Signature _____

Unambiguously circle your TA name and tutorial number

Brian Chan 8 23

Soomin Choi 1

Fazle Chowdhury 6 7 9 26

Ruixue (Vera) Dai 10 16

Daerian Dilkumar 14

Rui (Ray) Gao 3 4

Martin Ho 12

Kuni Ito 11 18

Jianheng (Season) Li 5 29

Jiawei (Ada) Lin 27

Tsun (Reynold) Lok 24

John Menacherry 17 21

Roleen Nunes 19 28

Zhengyuan (Kevin) Xie 8

Binya Xu 2

Sunghwan (Jacob) Yoo 20 22

Sheng Zang 25

Instructions

1. This test has 11 numbered pages. Check that all of these pages are included.
2. Put your letter answers to Part A Questions in the answer boxes at the top of Page 2. Put your solutions and/or rough work to Part B Questions in the answer spaces provided beneath each question. If you need extra space, use the back of a page or Page 11 and clearly indicate the location of your continuing work.
3. You may use **one** calculator of any make or model, except that it cannot receive/send data. All other electronic devices, scrap paper, notes, textbooks, pen/pencil carrying cases, foods, and hats (except for religious reasons) are forbidden at your workspace. You may have a drink that is not in any kind of paper cup or a container with a paper label.
4. You are encouraged to write your test in pen or other ink. If any questions (Part A or B) or rough work is displayed in pencil, then your entire test will be denied any remarking privilege.

Print letters for Part A (Multiple Choice Questions) in these boxes

1	2	3	4	5	6	7

Do not write anything in the boxes below

Info.	Part A
3	21

Part B

1	2	3	4	5	6
15	15	11	11	10	14

Total
100

Some formulas

$$S = P(1+r)^n \qquad S = Pe^{rt} \qquad \eta = \frac{p/q}{\frac{dp}{dq}}$$

$$S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$S = R\left[\frac{(1+r)^{n+1} - 1}{r}\right] - R \qquad A = R + R\left[\frac{1-(1+r)^{-n+1}}{r}\right]$$

Part A (Multiple Choice Questions) **Clearly print the letter of the answer you think is correct in the boxes at the top of Page 2.** Each correct answer earns 3 points and each incorrect or blank answer earns 0 points.

1. If $f(x) = \dfrac{5x^{3/5} + 6x + 4}{3\sqrt{x}}$ then $f'(1)$ equals

   (A) 1/3 (B) $-1/3$ (C) 2 (D) 1/2 (E) $-1/2$ (F) none of (A) - (E)

2. The present value of \$1,200 due in six years at a 2.5% nominal rate compounding quarterly is (rounded down to the nearest dollar)

   (A) \$1,033 (B) \$1,065 (C) \$983 (D) \$1,072 (E) none of (A) - (D)

3. What is the fewest whole number of months required for a principal to increase by at least 42% with interest of 3.2% APR compounding eight times annually?

   (A) 132 (B) 131 (C) 133 (D) 88 (E) 704 (F) 134 (G) 705

4. Let a and b be positive constants. If $u = \sqrt{3ax^2 + b}$ then $\dfrac{du}{dx}$ equals

(A) $\dfrac{2ax}{u}$ (B) $2axu$ (C) $\dfrac{3ax}{u}$ (D) $\dfrac{3a}{u}$ (E) $\dfrac{3ax}{\sqrt{u}}$ (F) none of (A) - (E)

5. The value of $\lim\limits_{x \to 1^-} \left(\dfrac{x^2 - 1}{|x - 1|} + x + 2 - e^{(x-1)} \right)$ is

(A) 3 (B) 2 (C) 1 (D) 0 (E) $1 - e$ (F) 4 (G) none of (A) - (F)

6. What (3-decimal approximate) APR of interest compounding continuously is equivalent to an effective rate of 2% ?

(A) 1.984% (B) 1.976% (C) 1.980% (D) 1.991%

(E) a percentage not in (A) - (D) (F) we cannot find this because no time period is given

7. If $y = x5^{(x^2+x)}$ then $\left.\dfrac{dy}{dx}\right|_{x=1}$ equals

(A) 35 (B) $25 + 30\ln(5)$ (C) 50 (D) $25 + 75\ln(5)$ (E) none of (A) - (D)

Make sure your answers are printed in the letter boxes at the top of Page 2

Part B (Full Solution Questions) Show all of your work. Answers/solutions will earn full points only if they are correct, complete, and sufficiently display relevant concepts from MATA32F.

1. The three parts of this question are independent of each other.

   (a) Let $y = \dfrac{2x+13}{4x+1}$. Evaluate $\dfrac{dy}{dx}$ at $x = 1$. [3 points]

   (b) Let $h(t) = (3t+2)(5t-20)$. Find all values of t such that $0 \leq h'(t) \leq 3$. [5 points]

   (c) Let $f(x) = \dfrac{x^2}{ln(x)+1}$. Find the slope-intercept equation of the tangent line at the point where $x = e$. Leave your answer in terms of mathematical constants. Do not use decimals. [7 points]

2. In all of this question $R > 0$ dollars are deposited into an ordinary annuity at the end of each month. Interest is 4.8% APR compounding monthly.

   (a) Find the effective rate of interest rounded up to 3 decimal places. [3 points]

   (b) If the annuity is empty to begin with, find the least whole number of years and months that it will take for the annuity to grow to $1,000R$ dollars. [8 points]

   (c) Which would you rather receive: an amount of $90R$ now or a five year term ordinary annuity with interest as described at the beginning of this question? [4 points]

3. Find each limit that exists. Use the ∞ and $-\infty$ symbol as appropriate. If a limit does not exist, justify why. Any solution obtained by l'Hopital's rule will not receive credit.

(a) $\lim_{x \to -5} \dfrac{x^2 + 25}{5 - x}$ [3 points]

(b) $\lim_{x \to \infty} \left[\dfrac{3 - 5x + 32x^3}{7 + 2x^2 - x^3} \right]^{1/5}$ [3 points]

(c) $\lim_{x \to 0} \dfrac{\sqrt[3]{1 + cx} - 1}{x}$ (c is a constant, $c \neq -1$) [5 points]

4. One debt of \$3,000 plus interest at 4% APR compounded quarterly is due at the end of 30 months from now. Another debt of \$7,000 plus interest at 3% APR compounded semiannually due at the end of five years from now. These two debts are to be repaid by two payments as follows:

   (i) a first payment at the end of one year from now;

   (ii) a second payment at the end of four years from now that is 70% of the first payment.

   Interest on all debts and all payments is 3.6% APR compounding monthly.

   Find the amount of the two payments. Carry at least five decimals in all of your calculations. Round your final answers up to the nearest dollar. A complete money-time diagram and equation of value are required for full points. [11 points]

5. In all of this question let $p = 1,200 - q^2$ be a demand function ($p > 0$ is unit price and $q > 0$ is quantity).

(a) Find the (point) elasticity of demand. [3 points]

(b) Find the value of q for which the demand has unit elasticity. [4 points]

(c) Find the marginal revenue when $q = 10$. [3 points]

6. The two parts of this question are independent of each other.

(a) A quadratic function $y = f(x)$ has a horizontal tangent line at the point $(1, -2)$ and its graph intersects the line $y = 3x + 1$ at the point where $x = 3$. Find the function $f(x)$.

[10 points]

(b) Suppose $y = h(x)$ and $x = g(t)$. Given that $g(3) = 5$, $g'(3) = -4$, $h(-4) = 1$, $h'(3) = 3$, $h(3) = 2$, and $h'(5) = -2$ evaluate $\dfrac{dy}{dt}$ at $t = 3$.

[4 points]

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