University of Toronto Scarborough
Department of Computer & Mathematical Sciences

Midterm Test

MATA32H – Calculus for Management I

Examiner: E. Moore

Date: February 27, 2017
Start time: 5:00pm
Duration: 110 minutes

1. [**5 points**]   You have decided to go "green" and have just purchased a good bike for $250 down and monthly payments of $125 for the next 6 months. Interest is $3.6\,\%$ APR compounded monthly and all payments are made at the end of the month. Calculate the price, rounded up to the nearest dollar, if you had paid the full amount now.

2. [**10 points**]   Find the following limits, if they exist, You may use the symbols ∞ or $-\infty$ when needed. Always provide justification when appropriate. (The use of l'Hôpital's Rule will earn no credit.)

   (a) $\displaystyle \lim_{x \to -4} \frac{x^3 + 4x^2}{x^2 + 2x - 8}$

   (b) $\displaystyle \lim_{t \to -\infty} \frac{3 - 2t - 2t^3}{7 - 5t^3 + 2t^2}$

   (c) $\displaystyle \lim_{x \to 2} f(x)$ when $f(x) = \begin{cases} 3x - 5 & , \text{ for } x \geq 2 \\ x^2 - x - 1 & , \text{ for } x < 2 \end{cases}$.

   Is f continuous at $x = 2$?

3. [**8 points**]   Let $f(x) = \dfrac{16 - 4\sqrt{x + 11}}{x - 5}$ where $x \geq 0$, $x \neq 5$ and $f(5) = -\dfrac{1}{2}$. Determine if f is continuous at $x = 5$. Explain.

4. [**12 points**] Let $f(x) = \dfrac{3\,x}{2\,x - 1}$.

   (a) Find the point(s) on the graph of $y = f(x)$ where the slope of the tangent line is $-\dfrac{1}{3}$.

   (b) Use the definition of derivative ("first principles") to find $f'(x)$.

5. [**10 points**]

   (a) Find $\dfrac{d\,y}{d\,x}$, in fully factored form, when $y = (2x - 3)^2\,(3x + 1)^4$.

   (b) Find $\dfrac{d\,y}{d\,x}$ when $y = \sqrt{\dfrac{x + 1}{x - 5}}$.

6. [**12 points**] Let $p = \sqrt{1600 - q^2}$ be a demand function for $0 \le q \le 40$.

    (a) Show that the (point) elasticity of demand is $\eta = -\dfrac{p^2}{q^2}$.

    (b) For $q = 15$, determine if the (point) elasticity of demand is elastic or inelastic.

    (c) Find the value(s) of q for which the (point) elasticity of demand is unit.

7. [**8 points**] Let $y = f(x)$ be defined implicitly by the expression

$$(x^2 + y^2)^2 = 4y^2.$$

    Find the equation of the tangent line at $(0, 2)$.

8. [**10 points**]   Let $f(x) = x^{1/3}(x - 2)$.

    (a) Find the critical points of $y = f(x)$, determine the intervals of increase and decrease and find relative extrema. (A sign chart is required.)

    (b) Explain if it is possible to find absolute extrema for $y = f(x)$ on $[-1, 4]$. Locate any absolute extrema that may exist.

9. [**15 points**]   A total debt of $5000 due 25 months from now and $3500 due 5 years from now is to be repaid by the following 3 payments:
    - The first payment is made 1 year from now,
    - The second payment is double the first and is made 3 years from now,
    - The third payment is the same as the second and made 4 years from now.
    Interest is at a nominal rate of 3.6% compounded monthly. Find the amount of each payment rounded up to the nearest dollar.
    (A complete answer requires a money-time line and an equation of value.)

10. [**15 points**]   A producer of a certain guidance system component has discovered that, when 10 units are produced, the average cost per unit is $50 and the marginal cost is $12.

    (a) Find the approximate cost to produce 11 units.

    (b) Find the marginal average cost to produce 10 units.

    (c) If the cost function is a quadratic and the fixed cost is $350, find the cost function.