University of Toronto Scarborough
Department of Computer & Mathematical Sciences

FINAL EXAMINATION

MATA32H – Calculus for Management I

Examiner: E. Moore

Date: April 5, 2017
Start Time: 2:00 PM
Duration: 3 hours

1. **[9 points]**

   (a) Find the exact value of $f'(1)$ where $f(t) = \dfrac{e^{t^2+1}}{\sqrt{t^2+1}}$.

   (b) Find y' where $y = \dfrac{x\,(1+x^2)^2}{\sqrt{2+x^2}}$.

2. **[12 points]**

   (a) Let $f(x) = \begin{cases} x^2 + 3x - 1 & \text{if} \quad x \geq 2 \\ k^2\,x^2 - x - 1 & \text{if} \quad x < 2 \end{cases}$. Find all values of k such that f is continuous at $x = 2$.

   (b) Find the point(s) on the curve $y^2 + x\,y - x^2 = 9$ where the tangent line is parallel to the line $2x + 4y + 3 = 0$.

3. **[4 points]** Given an effective rate r_e, what is the equivalent nominal rate r, if compounding is monthly.

4. **[5 points]** Explain mathematically why the function $f(x) = x^3 - 3x + 1$ has absolute extrema on $[-2, 3]$ and then find these extrema.

5. **[5 points]** In this question all money is in units of thousands of dollars.
   You have made an initial investment of 20 in a friend's business with guaranteed cash flows of 5 at the end of year 1, 6 at the end of year 2 and x at the end of year 3. Interest is $3\,\%$ APR compounded every 4 months. Find the value of x that will allow you to break even on the investment.
   (Your solution should include a money-time line.)

6. **[10 points]**

   (a) Suppose that the demand equation for a certain commodity is
   $$p = 4 - 0.0002\,q$$
   where q units are produced each day and p is the price of each unit. The cost of producing q units is $600 + 3\,q$. If the daily profit is to be as large as possible, find the number of units produced each day, the price of each and the daily profit.

(b) Suppose the government now imposes a \$0.20 tax on each unit produced. For maximal daily profit, how many units are now produced each day? What is the price of each unit and what is the daily profit?

7. [**12 points**] Sketch the graph of

$$f(x) = \frac{x}{(x-1)^3} \ .$$

A complete solution includes all calculations, sign charts and a fully labeled picture showing all the special features of this function.

8. [**8 points**] A marginal revenue function is given by

$$\frac{d\,r}{d\,q} = 3\,q^2 + 2\,e^{-q} + 3$$

for $q > 0$. Assuming $r(0) = 0$, find the demand function, $p = p(q)$.

9. [**10 points**] Let $\mathcal{R}$ be the region bounded by the two curves $x = y^2 - 1$ and $y = x - 1$.

   (a) Give a good labeled sketch of the region $\mathcal{R}$ and show a small rectangle that represents a typical "strip of area".

   (b) Find the area of $\mathcal{R}$.

10. [**11 points**]

    (a) Carefully state what is meant by the terms **antiderivative** and **indefinite integral**.

    (b) Find $\displaystyle\int x^2 \log_2 x \; dx$.

    (c) Find $\displaystyle\int \frac{e^{2x}}{1+e^{2x}} \, dx$.

11. [**11 points**]

    (a) Carefully state the **Fundamental Theorem of Integral Calculus**.

    (b) Evaluate $\displaystyle\int_{-1}^{0} \frac{x^2+4x-1}{x+2} \, dx$

    (c) Evaluate $\displaystyle\int_{1}^{2} \left(2\sqrt{x} - \frac{3\,x}{\sqrt{2x^2+1}} \right) \, dx$.

12. [**8 points**] The demand equation for a product is

$$p = 0.01q^2 - 1.1\,q + 30$$

and the supply equation is

$$p = 0.01q^2 + 8 \ .$$

Determine the consumers' surplus and producers' surplus when market equilibrium has been established.