University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION
MATA32 - Calculus for Management I

Examiner: R. Grinnell

Date: April 21, 2012
Time: 2:00 pm
Duration: 3 hours

Provide the following information:

Lastname (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number : ______________________________

Signature: ______________________________

Read these instructions:

1. This examination has 13 numbered pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.
2. If you need extra answer space, use the back of a page or page 13. Clearly indicate the location of your continuing work. You may write in pencil, pen, or other ink.
3. You may use one standard hand-held calculator (graphing capability is permitted). All other electronic devices (e.g. cell phone, smart phone, i-pod), extra paper, notes, textbooks, and backpacks are forbidden at your workspace.
4. If you have brought a cell/smart phone into the exam room, it must be turned off and left at the front.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10	11	12

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	**TOTAL**
42	15	16	18	15	14	16	14	150

The following may be helpful:

$$S = P(1+r)^n \qquad S = Pe^{rt} \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$S = R\left[\frac{(1+r)^{n+1} - 1}{r}\right] - R \qquad A = R + R\left[\frac{1-(1+r)^{-n+1}}{r}\right]$$

Profit = Revenue - Cost $\qquad NPV = (\sum PV) - $Initial $\qquad \eta = \dfrac{p/q}{dp/dq}$

Part A: 12 Multiple Choice Questions For each of the following, **clearly print the letter of the answer you think is most correct in the boxes on page 1**. Each right answer earns 3.5 points and no answer/wrong answers earn 0 points. No justification is required.

1. If $y = x^2 \, ln(x)$ then $y'(e)$ equals

   (A) $3e$ (B) $2e^2 + e$ (C) $2e + e^2$ (D) $2 + e$ (E) none of (A) - (D)

2. The slope of the curve $y^2 + y + 6ln(x) = 0$ at $(1, -1)$ is

   (A) 2 (B) 3 (C) -6 (D) 6 (E) a number not in (A) - (D)

3. The value of $\int_0^1 (3+6x)\sqrt{2x+2x^2}\, dx$ is

   (A) 12 (B) 8 (C) 6 (D) 18 (E) a number not in (A) - (D)

4. If $m > 0$ is a constant, then the value of $\displaystyle\lim_{x\to\infty}\left(\frac{x^3}{x^2 - mx} - x\right)$ is

(A) m (B) $-m$ (C) m^2 (D) $1 - m$ (E) nonexistent
(F) a value not in (A) - (D)

5. If $y = e^{1/x}$ then $\dfrac{dy}{dx}$ equals

(A) yx^{-2} (B) $-yx^{-2}$ (C) yx^{-1} (D) $-yx^{-1}$ (E) none of (A) - (D)

6. If a manufacturer's total-cost function is given by $c = 12q^2 + 3,888$ where $q \geq 0$ is the number of units produced, then the minimum average cost per unit is

(A) 18 (B) 237.56 (C) 412 (D) 432 (E) a number not in (A) - (D)

7. The least possible whole number of months for an amount to increase by 50% at 5% APR is

(A) 110 (B) 106 (C) 98 (D) 94 (E) 93 (F) none of (A) - (E)

8. If c is a negative constant and $h(x) = x\,e^{cx}$ then h has

(A) a relative maximum at $x = 1/c$ (B) a relative maximum at $x = -1/c$

(C) a relative minimum at $x = 1/c$ (D) a relative minimum at $x = -1/c$

(E) none of properties (A) - (D)

9. If $p = -4q + 152$ is a demand function where p is the unit price and $q \in (0, 35)$ is the quantity, then we have unit elasticity at

(A) $q = 15$ (B) $q = 17.5$ (C) $q = 19$ (D) $q = 21.5$

(E) a value of q not in (A) - (D) (F) no value of q

10. Let $\alpha \in (0, 1)$ be a constant and let $c = q^{\alpha}$ be a cost function where $q > 0$ is quantity. Consider the following four statements that refer to c:

(i) the marginal cost function is increasing on $(0, \infty)$

(ii) the average cost function is increasing on $(0, \infty)$

(iii) the marginal cost function is decreasing on $(0, \infty)$

(iv) the average cost function is decreasing on $(0, \infty)$

Which of the above statements are true?

(A) (i) and (ii) (B) (i) and (iv) (C) (ii) and (iii) (D) (iii) and (iv)

11. Let h be a continuous function on the closed interval $[0,5]$ such that $\int_0^5 h(x)\,dx = 8$, $\int_3^5 h(x)\,dx = 1$, and $\int_0^4 h(x)\,dx = 4$. The value of $\int_3^4 h(x)\,dx$ is

(A) 3 (B) 1 (C) -4 (D) -3 (E) not in (A) - (D)

(F) unable to be found because we do not have enough information about the integrals of h

12. Exactly how many of the following four statements are always true?

- If for all real x, $F_1(x)$ and $F_2(x)$ are antiderivatives of a function $f(x)$, then $F_1'(x) = F_2'(x)$.
- The definite integral of a function g is a function G such that $\int g(x)\,dx = G(x) + C$ where C is an arbitrary constant.
- Given a function $h(x)$ that is continuous on the closed interval $[1,4]$ we have that $\int_1^4 h(x)\,dx$ is defined to be $H(4) - H(1)$ where H is any antiderivative of h.
- If n is an integer then $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$ where C is an arbitrary constant.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

BE SURE YOU HAVE PUT YOUR LETTERS IN THE BOXES ON PAGE 1

Part B: 7 Full Solution Questions Write clear, full solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32.

1. (a) Find the equation of the tangent line to $y = \dfrac{4e^x}{x+2}$ at the point on the curve where $x = 0$. Give your answer in slope-intercept form. [7 points]

   (b) Assume a function f is defined for all real numbers x and $f(x) = c^2x + 1$ if $x < 2$ and $f(x) = 2cx^2 - 5$ if $x > 2$ (c is a constant). Find the values of c that make f continuous at $x = 2$. For each value of c, find $f(2)$. [8 points]

2. (a) Find the value of the constant K so that the function $g(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + K$, where $x \in [0, 1]$, has an absolute minimum value of 3 . Sufficiently justify your answer.

[9 points]

(b) In this question assume all money is in units of \$-thousands. Suppose an initial investment of 37 in a small home-based business guarantees the following cash flows:

Year	1	2	3	4
Cash Flow	8	13	15	F

Interest is 2% APR compounding annually and all cash flow payments are made at the end of each year. Find the smallest value of F above (rounded up to the nearest dollar) so that the business investment will be profitable. [7 points]

3. (a) Find $\displaystyle\int \frac{7\left(1+\sqrt{x}\right)^{3/4}}{\sqrt{x}}\,dx$ [6 points]

(b) Evaluate $\displaystyle\int_1^e x^2 ln(x)\,dx$ [8 points]

(c) State a region $\mathcal{R}$ and explain why the integral in (b) above represents the area of $\mathcal{R}$. [4 points]

4. In all of this question we consider approximating the number $r = \sqrt{14}$ by using Newton's Method (NM) with the function $g(x) = x^2 - 14$.

(a) Find the positive integer a such that $r \in [a, a+1]$. Use a theorem from our course to sufficiently justify your answer. [5 points]

(b) Show that NM applied to g gives the formula $x_{n+1} = \dfrac{x_n}{2} + \dfrac{7}{x_n}$ [5 points]

(c) Let x_1 be defined such that $|g(x_1)|$ is the smaller of $|g(a)|$ and $|g(a+1)|$ Use part (b) to find x_2 and x_3 as rational numbers, not decimals. [5 points]

5. Let $y = f(x) = 4x^{3/5} - x^{8/5}$ [14 points]

(a) State the domain of f and the x-intercepts.

(b) Find all critical values of f.

(c) State the interval(s) where f is increasing and decreasing, and state all relative extrema.

(d) Use the full space below to sketch an accurate graph of $y = f(x)$. Clearly show all of the features in parts (a) - (c). Points are awarded for accuracy and neatness.

6. In all of this question let $y = c(q)$ be a total-cost function where $q > 0$ is the number of units produced. Assume the marginal cost function is $M(q) = 24 - \dfrac{384}{(q+1)^2}$ and the average cost when 3 units are produced is \$45 per unit.

(a) Find the total-cost function. [8 points]

(b) Use either the 1^{st} or 2^{nd} derivative test to determine whether the total-cost function has a relative extrema and state the type of extrema (i.e. maximum or minimum). If there is a relative extrema, where does it occur? Is it an absolute extrema? Justify your answer. [8 points]

7. (a) For $x > 0$, let $f(x) = \left(\frac{1}{x}\right)^x$. Find the exact value of $f'(e)$ and simplify your answer. ("exact value" means no decimals). [7 points]

(b) Let $A = \frac{R}{r}\left[1 - (1+r)^{-n}\right]$ represent the amount of an ordinary annuity considered as a function of the variable n. Show that $\frac{dA}{dn} + A\,ln(1+r) = B\,ln(1+r)$ where $B = \frac{R}{r}$ [7 points]

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