*** Sorry...no solutions are posted ***

University of Toronto at Scarborough

Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA32F - Calculus for Management I

Examiners: R. Grinnell
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Date: Dec 13, 2017
Time: 7:00 pm
Duration: 2 hr 50 min

LAST NAME (PRINT BIG) ______________________________

Given Name(s) (PRINT BIG) ______________________________

Student Number ______________________________

Signature ______________________________

Instructions

1. This examination has 12 numbered pages. Check that all of these pages are included.
2. If you need extra answer space, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of your continuing work.
3. The following are forbidden at your work space: calculators, smart/cell phones, all other electronic devices, extra paper, notes, textbooks, opaque carrying cases, food, and drinks in paper cups/boxes or with a label.
4. You may write in pencil, pen, or other ink.

Print letters for the Multiple Choice Questions in these boxes

1	2	3	4	5	6	7	8	9

Do not write anything in the boxes below

A	1	2	3	4	5	6	7	8	**TOTAL**
27	18	15	12	17	18	12	16	15	150

Some formulas

$$A = R\Big[\frac{1-(1+r)^{-n}}{r}\Big] \qquad S = R\Big[\frac{(1+r)^{n}-1}{r}\Big] \qquad S = P(1+r)^n$$

Part A (Multiple Choice Questions) For each of the following, **clearly print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 3 points and no answer/wrong answers earn 0 points. No justification is required.

1. The exact value of $\int_0^1 4x(x^2+1)^5\,dx$ is

   (A) 21 (B) 42 (C) 64/3 (D) 41/3 (E) 23

2. If $f(x) = 4(2^x)$ and n is a positive integer, then $f^{(n)}(1)$ equals

   (A) $2(8^n)$ (B) $8(ln(2))^n$ (C) $(8ln(2))^n$ (D) 8^n (E) $[4ln(2)](2^n)$

3. The area of the region lying between the curves $y = 3$ and $y = x^2 - 2x$ where $0 \leq x \leq 3$ is

   (A) 26/3 (B) 14/3 (C) 16/3 (D) 6 (E) 9

4. If $q \geq 0$ is quantity and cost is given by $c = q^3 - 12q^2 + 300$, then the marginal cost is increasing on the interval

   (A) $(4, \infty)$ (B) $(8, \infty)$ (C) $(0, 8)$ (D) $(0, 4)$ (E) $(0, \infty)$

5. If $u'(t) = 3t - 2$ and $u(4) = 22$ then $u(0)$ equals

(A) 4 (B) 6 (C) 8 (D) 14/3 (E) 14

6. If $f(x) = \left(\frac{e}{x}\right)^x$ then $f'(1)$ equals

(A) -1 (B) 1 (C) 0 (D) $-e$ (E) e

7. The APR $a\%$ compounding every three months that is equivalent to a given effective rate of r_e is

(A) $3(\sqrt[3]{1+r_e}-1)$ (B) $\frac{1}{4}\sqrt[4]{1+r_e}-1$ (C) $4(\sqrt[4]{1+r_e}+1)$ (D) $4(\sqrt[4]{1+r_e}-1)$

8. If $c < 0$ is a constant and $h(x) = cxe^{cx}$, then h has a

(A) local maximum at $x = -1/c$ (B) local minimum at $x = -1/c$
(C) local minimum at $x = 1/c$ (D) local maximum at $x = 1/c$

9. Exactly how many of the following statements are always true?

- The definite integral of a continuous function h over $[a, b]$ is a function obtained by taking the limit of a Riemann sum.
- If a is a real constant, then $\int x^a\,dx = \frac{x^{a+1}}{a+1} + C$.
- If c is a critical number of a continuous function f and $f(c) > 0$, then f has a relative extrema at c.
- If g is a continuous function and $g''(c)$ is equal to 0 or is undefined, then g has an inflection point at c.

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

Be sure you have printed the letters for your answers in the boxes on the first page

Part B (Full-Solution Questions) Write clear solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32F. Show all work.

1. The parts of this question are independent of each other.

   (a) Find the exact value of $f'(1)$ where $f(x) = (4x^2 + 5)^x + log_2(x)$. [9 points]

   (b) Assume $h'(x) = 9xe^{3x}$ and $h(0) = 4$. Find $h(x)$. [9 points]

2. The parts of this question are independent of each other.

(a) For $q \in [100, 400]$, let the demand for a certain product be $p = \sqrt{500 - q}$. Here q is quantity and p is unit price. Find the value of q that maximizes the revenue and calculate the corresponding maximum revenue. [9 points]

(b) Let $x > 0$ represent quantity and let $c = Ax^2 + Bx + K$ be a cost function where A, B, K are positive constants. Show that the marginal average cost function is increasing.

[6 points]

3. An box with a square base and no top is to be constructed from $A\ m^2$ of material where A is a positive constant. What should the dimensions of the box be if the volume is to be a maximum? What is the maximum volume? Provide an appropriate diagram and sufficiently justify your solution.

[12 points]

4. Evaluate the following two definite integrals.

(a) $\displaystyle\int_0^1 \frac{1}{3+e^{-2x}}\,dx$ [8 points]

(b) $\displaystyle\int_1^4 \sqrt{x}\,ln(x^9)\,dx$ [9 points]

5. The parts of this question are independent of each other.

(a) Let $x_1 = 2$ and select the appropriate function for Newton's method to approximate the number $\sqrt[3]{10}$. Calculate x_2. [6 points]

(b) Let $A(n) = \frac{R}{r}[1 - (1+r)^{-n}]$ be the present value of an ordinary annuity as a function of n. Show that $\frac{dA}{dn} + Ac = Bc$ where $B = \frac{R}{r}$ and $c = ln(1+r)$. [8 points]

(c) Carefully state what is meant by the term **antiderivative**. [4 points]

6. Let $f(x) = e^2x^2e^x$. Find the intercepts of f, the intervals of increasing/decreasing, and where f has relative extrema and the kind of extrema. Use this information to make a good sketch of the graph of $y = f(x)$. Points are awarded for accuracy and neatness. [12 points]

7. In all of this question let $\mathcal{C}$ represent the curve $x = y^2 - 2y + 3$.

(a) Find an equation of the tangent line to the curve $\mathcal{C}$ at the point of intersection of $\mathcal{C}$ and the line $y = 2$. [6 points]

(b) Let $\mathcal{R}$ represent the region bounded between the curve $\mathcal{C}$ and the curve $x = -y^2$ where $0 \leq y \leq 2$. Calculate the area of $\mathcal{R}$. Give a neat, labeled diagram of $\mathcal{R}$ and show a typical "strip of area". [10 points]

8. The parts of this question are independent of each other.

(a) Let $q \geq 0$ represent quantity. A demand function is $p = D(q) = -0.1q^2 + 90$ and a supply function is $p = S(q) = 0.2q^2 + q + 50$. Calculate the consumers' surplus when market equilibrium has been established. [10 points]

(b) Evaluate the limit: $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{\sqrt{k}}{n\sqrt{n}} + \frac{k}{n^2} \right)$. [5 points]

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