***** Sorry...no solutions will be posted*****

University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION
MATA32F - Calculus for Management I

Examiners: N. Cheng
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Date: December 6, 2016
Time: 2:00 pm
Duration: 170 minutes

LAST NAME (CAPITALIZE) ______________________________

Given Name(s) (PRINT BIG) ______________________________

Student Number ______________________________

Signature ______________________________

Read these instructions

1. This examination has 11 numbered pages. At the beginning of the exam, check that all of these pages are included.
2. If you need extra space for any question, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of your continuing work. Show all work.
3. The following are forbidden at your work space: calculators, i-pads, smart/cell phones, all other electronic devices (e.g. electronic translation/dictionary devices), extra paper, notes, textbooks, opaque carrying cases, food, and drinks in paper cups/boxes or with a label.
4. You may write in pencil, pen, or other ink. You may use correction fluid/tape.

Do not write anything in the boxes below

1	2	3	4	5	6	7	8	9	**TOTAL**
17	20	17	17	21	12	13	13	20	150

Some Formulas

revenue = (unit price) × (quantity) $\qquad S = Pe^{rt} \qquad S = P(1+r)^n \qquad NPV = (\sum PV) - P$

Exam Instructions Give clear, complete answers in the spaces provided. Full points are awarded only for answers that are correct, complete, and display sufficient justifications and concepts from MATA32F. Show all work.

1. (a) Find $f'(2)$ where $f(x) = (x^3 - 3x)^5$. [4 points]

   (b) Find $h(16)$ where $h'(t) = \dfrac{3}{\sqrt{t}}$ and $h(1) = 5$. [4 points]

   (c) Find the slope of the curve $5xy + y^3 - 4x = 6x^2 + 8$ at the point $(0, 2)$. [5 points]

   (d) Evaluate $\int [(x + \sqrt{x})^2 + 8e^{2x}]\, dx$. [4 points]

2. (a) Find $g''(1)$ if $g(x) = e^{x^2} + x^3$.

[5 points]

(b) Evaluate $\int_1^e \frac{ln(x^2)}{x}\, dx$. [5 points]

(c) If $q > 0$ is quantity and $\overline{c} = 0.4q + 12 + 20\left(\frac{1 + 15\, ln(q)}{q}\right)$ is an average cost function, find the marginal cost when $q = 50$. [5 points]

(d) Find the interval(s) on which the function $f(x) = x^4 - 2x^3 - 12x^2$ is concave up.

[5 points]

3. (a) Find $y'(1)$ where $y = 8(ex)^{\sqrt{x}}$. [6 points]

(b) Assume Newton's method is used to approximate the number $\sqrt[4]{36}$. If $x_1 = 2$, calculate x_2 as a simplified fraction. [6 points]

(c) Let $f(x) = \begin{cases} -x^2 & if \quad -1 \leq x \leq 0 \\ 2x & if \quad 0 \leq x \leq 1 \end{cases}$ Evaluate $\int_{-1}^{1} f(x)\, dx$ as a simplified fraction.

[5 points]

4. (a) Let q be quantity where $1 \leq q \leq 6$. Assume demand (i.e. unit price) is $p = 192 - q^2$. Find the value of q that gives the maximum revenue and find the maximum revenue.

[7 points]

(b) What annual rate of interest compounded continuously over eight years is equivalent to a rate of 3% APR compounding every four months for eight years? [4 points]

(c) An initial investment of 40 in a business guarantees cash flows of 4 at the end of Year 1, 7 at the end of Year 2, 12 at the end of Year 3, and L at the end of Year 6. Interest is 6% APR compounding at the end of every other month. Solve for the quantity L so that the business is neutral (i.e. no profit and no loss). Draw a small money-time diagram as part of your solution. [6 points]

5. (a) Evaluate $\int_0^1 \frac{e + e^{1+x}}{e^x}\, dx$. [6 points]

(b) Find $\int (6x^5 + 12x^2)\sqrt{2 + x^3}\, dx$. [6 points]

(c) Evaluate $\int_1^e \frac{ln(t)}{\sqrt{t}}\, dt$. [9 points]

6. Draw an accurate, neat graph of a continuous function $y = f(x)$ that has all of the properties below. Note that there are many functions with these properties. Points are awarded for neatness and accuracy. Use the space beneath your graph to display any rough work. Your axis should be numbered -4 to 6 on the x-axis and -3 to 4 on the y-axis. Your graph should occupy most of the width of the page. [12 points]

   - $f(-2) = -2$ and $f(0) = 3$.
   - $f(x) > 1$ when $x > 4$.
   - $f'(x)$ has the same sign (i.e. positive, negative, zero) as the function $g(x) = x^2 - 3x$ when $-1 < x < 4$.
   - f is not differentiable at $x = -2$.
   - $f''(x) < 0$ when $x < -2$ or when $x \in (-2, 0)$.
   - $\lim_{x \to -\infty} f(x) = 1$.
   - $\lim_{x \to \infty} f(x) = 1$.

7. A rectangular plot of land has an area of $A > 0$ square metres. The length of the plot is drawn horizontally and the width is drawn vertically. Fencing that costs \$$k$ per metre is required on one side of length and both sides of width. Fencing that costs \$$n$ per metre is used for the remaining side of length. Find the length and width of the plot that makes the total fence cost as small as possible. Your answer should be expressed in terms of A, k and n. Make a labeled diagram, show all work, and give sufficient justification. [13 points]

8. A demand function is $p = f(q) = -q^2 + 100$ where p is unit price in dollars and q is quantity. A supply function is $p = g(q) = \frac{32}{3}q$. Assume $p \geq 0$ and $0 \leq q \leq 10$. Find the equilibrium point (Hint: $\sqrt{4624} = 68$). Draw the graphs of demand and supply on the same axis and show the area that represents consumers's surplus. Calculate the consumers' surplus under market equilibrium. [13 points]

9. (a) Let $\mathcal{R}$ be the region bounded by $y = x^2$ and $y = 9$. Find the number c so that the horizontal line $y = c$ divides the region $\mathcal{R}$ into two regions where the area of the lower region is twice the area of the upper region. Make a small labeled sketch of $\mathcal{R}$. [9 points]

(b) Give a complete statement of the **definition** of the definite integral of a continuous function $y = f(x)$ over $[a, b]$. [5 points]

(c) Evaluate $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{\sqrt{k}}{n\sqrt{n}} + \frac{1}{k+n}\right]$. [6 points]

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