*** Sorry...solutions will not be provided ***

University of Toronto at Scarborough

Department of Computer and Mathematical Sciences

FINAL EXAMINATION

MATA32F - Calculus for Management I

Examiners: R. Haslhofer, R. Grinnell, E. Moore

Date: December 13, 2015
Time: 2:00 pm
Duration: 2 hours and 50 minutes

Provide the following information

LAST NAME (CAPITALIZE) ______________________________

Given Name(s) (PRINT BIG) ______________________________

Student Number ______________________________

Signature ______________________________

Read these instructions

1. This examination has 12 numbered pages. At the beginning of the exam, check that all of these pages are included.
2. If you need extra space for any question, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of your continuing work.
3. The following are forbidden at your work space: calculators, i-pads, smart/cell phones, all other electronic devices (e.g. electronic translation/dictionary devices), extra paper, notes, textbooks, opaque carrying cases, food, and drinks in paper cups/boxes or with a label.
4. You may write in pencil, pen, or other ink.

Do not write anything in the boxes below

1	2	3	4	5	6	7	8	9	10	**TOTAL**
16	15	16	15	14	11	16	17	14	16	150

The following may be helpful

$$\eta = \frac{p/q}{dp/dq} \qquad S = P(1+r)^n \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Exam Instructions Write clear neat solutions in the answer spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts and knowledge from MATA32F.

1. (a) Let $f(x) = 2(5^x)$. Calculate $f^{(2)}(1)$. [3 points]

   (b) Let $u'(t) = 3t^2 - 6\sqrt{t}$ and $u(1) = -4$. Find $u(4)$. [5 points]

   (c) Assume Newton's method is used to approximate the number $\sqrt{20}$ and let $x_1 = 4$. Calculate x_2. [5 points]

   (d) Let an effective rate be r_e and let r be the APR compounded quarterly that is equivalent to r_e. Solve for r in terms of r_e. [3 points]

2. The parts of this question are independent of each other.

(a) Assume y is defined implicitly as a function of x by the equation $2\sqrt{y} + ln(xy^2) = 1$. Solve for x when $y = 1$ in this equation, and then evaluate $\dfrac{dy}{dx}$ at the point $(x, 1)$.

[7 points]

(b) Let $y = \sqrt{\dfrac{2x(x-2)}{x^2+9}}$. Calculate the exact value of $y'(4)$ and state your answer as a fraction in lowest terms.

[8 points]

3. Evaluate the following.

(a) $\displaystyle\int \frac{x^2 + 2x}{\sqrt{1-x}}\,dx$ [8 points]

(b) $\displaystyle\int_0^1 ln(2x+1)\,dx$ [8 points]

4. In all of this question let $q > 0$ represent quantity (i.e. number of units) of some product and let $p \geq 0$ represent unit price (in \$ per unit). Assume a demand function $p = 400 - 50q + \dfrac{2250}{q}$ and an average cost function $\overline{c} = \dfrac{800\, ln(q+5)}{q}$.

(a) State the profit function. [4 points]

(b) Find the quantity that maximizes profit and calculate the maximum profit. Sufficiently justify your answer. [11 points]

5. (a) Assume $p = f(q) = (q+3)e^{-q}$ is a demand function where $q > 0$ is quantity. Find all values of q such that the (point) elasticity of demand is elastic.

[9 points]

(b) In this question assume all money is in units of $\$1,000$'s. An initial investment of 47 in a small business is guaranteed to produce the following cash flows:

5 at the end of Year 1; 14 at the end of Year 2; F at the end of Year 4.

Interest is 4% APR compounding quarterly. The letter F represents the cash flow payment amount that makes the business "neutral" (i.e. the business has no loss and no profit). Solve for F.

[5 points]

6. In all of this question let $\mathcal{R}$ represent the region in the x, y-plane that is bounded between the curves $y = ln(x)$ and $x = y^2 + 2y + 3$ where $-2 \leq y \leq 0$.

(a) Make a good, labeled sketch of the region $\mathcal{R}$. [4 points]

(b) Find the area of $\mathcal{R}$. [7 points]

7. Let $f(x) = x - 3x^{1/3}$. State all intercepts of the function f. Find all intervals where f is increasing/decreasing and all intervals where f is concave up/concave down. State the local extrema of f and points of inflection. Make an accurate sketch of the graph of $y = f(x)$ that shows all of the features above.

[16 points]

8. In all of this question let $f(x) = \begin{cases} x^2 - 4x + 5 & if \quad x \geq 2 \\ c^2 x - c & if \quad x < 2 \end{cases}$ The letter c is a constant.

(a) Find all values of c that makes the function f continuous at $x = 2$. Sufficiently justify your answer. [7 points]

(b) On the same axis, make a sketch of the graphs of the function f for the values of c you found in Part (a). A tangent line to the curve $y = f(x)$ has y-intercept of -4. Find the equation of this tangent line.

[10 points]

9. Let $a > 0$ be a constant. Consider a box that has a square base, four walls, and a top. The cost of the base material is $\$3a/m^2$ and the cost of the other material is $\$1a/m^2$. The total cost of all materials (and only materials) used to make the box is fixed at \$108 and all of this amount is used. Find, in terms of the constant a, the largest possible volume of such a box and its dimensions. Sufficiently justify your solution. [14 points]

10. In all of this question let $f(x) = -x^2 + 4x$.

(a) Make a small sketch of the region bounded by the x-axis and the curve $y = f(x)$ and then use the Fundamental Theorem of Calculus to find its area. [5 points]

(b) Use the definition of the definite integral to find the area of the region in Part (a). Show all of your work. [11 points]

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