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University of Toronto at Scarborough

Department of Computer and Mathematical Sciences

FINAL EXAMINATION
MATA32F - Calculus for Management I

Examiners: R. Buchweitz
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Date: December 8, 2014
Time: 9:00 am
Duration: 3 hours

Provide the following information:

LAST NAME (PRINT BIG): ______________________________

Given Name(s) (PRINT BIG): ______________________________

Student Number: ______________________________

Signature: ______________________________

Read these instructions:

1. This examination has 13 numbered pages. At the beginning of the exam, check that all of these pages are included.
2. If you need extra space for any question, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of your continuing work.
3. The following are forbidden at your work space: calculators, i-pads, smart/cell phones, all other electronic devices (e.g. electronic translation/dictionary devices), extra paper, notes, textbooks, opaque carrying cases, food, and drinks in paper cups/boxes or with a label.
4. You may write in pencil, pen, or other ink.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	8	**TOTAL**
30	16	15	16	13	16	16	13	15	150

The following may be helpful:

$$\eta = \frac{p/q}{dp/dq} \qquad S = P(1+r)^n \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$r_e = \left(1+\frac{r}{n}\right)^n - 1 \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Part A (Multiple Choice Questions) For each of the following, **clearly print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 3 points and no answer/wrong answers earn 0 points. No justification is required.

1. If $h'(x) = \sqrt{2x+7}$ and $h(1) = 8$ then $h(-3)$ equals

   (A) $2/3$ (B) $-28/3$ (C) 16 (D) $-2/3$ (E) none of (A) - (D)

2. If $u = -(4x^2+1)^{-1/2}$ then $\frac{du}{dx}$ equals

   (A) $4xu^{-3}$ (B) $-4xu^3$ (C) $2xu^{-3}$ (D) $4xu^{-3/2}$ (E) $-4xu^{-3}$

3. Given an effective rate r_e, the equivalent nominal rate of r compounding quarterly is given by

   (A) $4\left(\sqrt[4]{1+r_e}\right)$ (B) $3\left(\sqrt[4]{1+r_e}-1\right)$ (C) $4\left(\sqrt[4]{1+r_e}-1\right)$ (D) $4\left(\sqrt[4]{1-r_e}+1\right)$

4. The area of the region bounded by the x-axis and the curve $y = x^2 - 3x$ is

(A) $9/2$ (B) 3 (C) $19/2$ (D) $-9/2$ (E) $27/4$ (F) none of (A) - (E)

5. If $p = -3q + 150$ is a demand function where the quantity q satisfies $0 < q < 50$, then we have unit elasticity at (A) $q = 48$ (B) $q = 30$ (C) $q = 25$

(D) $q = 5$ (E) a value of q not in (A) - (D) (F) no value of q

6. If $a < 0$ is a constant and $f(x) = xe^{-\frac{x}{a}}$, then we may conclude that f has a

(A) local maximum at $x = a$ (B) local minimum at $x = a$

(C) local maximum at $x = -a$ (D) local minimum at $x = -a$

7. The exact value of $\displaystyle\int_1^{e^2} \frac{[ln(x)]^2}{2x}\,dx$ is

(A) $\dfrac{e^6 - 1}{6}$ (B) $4/3$ (C) 1 (D) $8/3$ (E) $\dfrac{e^6 - 1}{3}$ (F) none of (A) - (E)

8. Assume your annual salary increases by a fixed percentage at the end of each year and that it takes eight years for your salary to double. The percentage rate r (as a decimal) of your annual salary increase is given by

(A) $2^{\frac{1}{8}} - 1$ (B) $8ln(2) - 1$ (C) $e^{\frac{ln(2)}{8}}$ (D) $8\left(2^{\frac{1}{8}} - 1\right)$

9. If $y = 2(5^x)$ and n is a positive integer, then $y^{(n)}(1)$ equals

(A) $10\left(ln(5)\right)^{-n}$ (B) $\left(2ln(5)\right)5^n$ (C) $\left(10\,ln(5)\right)n$ (D) $10\,ln(5^n)$ (E) $10\left(ln(5)\right)^n$

10. Exactly how many of the following statements are always true?

(i) If f is continuous at 0 and $\int_{-1}^{1} f(x)\,dx > 0$, then f is differentiable at 0.

(ii) The definite integral of a continuous function h over $[a, b]$ is a function H such that $\int_a^b h(x)\,dx = H(b) - H(a)$.

(iii) A continuous function g has a local minimum at a number c if and only if c is a critical number of g.

(iv) If a is a critical number of a polynomial, then the 2nd derivative test is always preferable over the 1st derivative test when checking for extrema at a.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Be sure you have printed the letters for your answers in the boxes on the first page

Part B (Full-Solution Questions) Write clear solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32F.

1. The parts of this question are independent of each other.

   (a) Find the exact value of $y'(1)$ where $y = \dfrac{x^4 e^{(x+x^2)}}{\sqrt{x^2+3}}$. [8 points]

   (b) Find the cubic polynomial $y = Q(x)$ having all of these properties:
   $Q(1) = 31, \quad Q'(1) = 7, \quad Q''(1) = 24,$ and $\quad Q'''(1) = 24.$ [8 points]

2. The parts of this question are independent of each other.

(a) Find the absolute extrema of the function $f(x) = x^3 - 12x + 1$ when $x \in [-1, 3]$ and state where these extrema occur. Sufficiently justify your answer. [8 points]

(b) Find $\displaystyle\int \frac{1}{\sqrt{x}\,(x + 2a\sqrt{x} + a^2)}\, dx$ where $a > 0$ is a constant. [7 points]

3. In all of this question let $q > 0$ represent quantity (i.e. number of units) and $p > 0$ represent unit price (i.e. price per unit). Assume a demand function $p = \dfrac{50}{\sqrt{q}}$ and an average cost function $\bar{c}(q) = \dfrac{1}{4} + \dfrac{100}{q}$.

(a) Given that "profit = revenue - cost" find the profit function, $F(q)$. [4 points]

(b) Find the value of q that maximizes total profit and state the maximum profit. Sufficiently justify that your value actually maximizes the total profit function. [9 points]

(c) Verify that "marginal revenue" = "marginal cost" at the value of q that maximizes total profit.

[3 points]

4. (a) Give a good, labeled sketch of the region $\mathcal{R}$ that is bounded by the two curves $x = y^2 + 1$ and $x = -(y-1)^2$ and the two lines $y = -1$ and $y = 1$. [5 points]

(b) Find the area of $\mathcal{R}$. [8 points]

5. Evaluate the following two definite integrals.

(a) $\int_0^1 x^3 \sqrt[4]{1+15x^4}\, dx$. Simplify your answer as much as you can.

[7 points]

(b) $\int_0^2 2xe^{-3x}\, dx$. Simplify your answer as much as you can.

[9 points]

6. In all of this question let $y = h(x) = x^4 + 2x^3 - 1$.

(a) Find the intervals where h is (i) increasing/decreasing and (ii) concave up/down.

[6 points]

(b) Find all relative extrema and point(s) of inflection of h. Sufficiently justify your answer.

[4 points]

(c) Give an accurate, labeled sketch of the graph of $y = h(x)$. Be sure to include all features you found in Parts (a) and (b).

[6 points]

7. A square sheet of cardboard of side length a is used to make an open box by cutting equal squares from the four corners and folding up the sides. What size squares (in terms of a) should be cut out of the corners to obtain a box of maximum possible volume? What is the maximum possible volume (in terms of a)? Show all work and justification in your solution.

[13 points]

8. The parts of this question are independent of each other.

(a) Find the equation of the tangent line to the curve $y = x^3 - 3x^2 + 5x$ that has the smallest possible slope. Give your answer in "slope-intercept" form. Sufficiently justify your work. [8 points]

(b) Let S represent the amount of an ordinary annuity as a function of n, the number of compounding periods. Show that $\dfrac{dS}{dn} = \alpha(K+S)$ where $K = \dfrac{R}{r}$ and $\alpha = ln(1+r)$. [7 points]

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