University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION
MATA32F - Calculus for Management I

Examiners: R. Buchweitz
R. Grinnell
S. Rayan

Date: December 11, 2013
Time: 9:00 am
Duration: 3 hours

Provide the following information:

Lastname (PRINT): ____________________

Given Name(s) (PRINT): ____________________

Student Number: ____________________

Signature: ____________________

Read these instructions:

1. This examination has 13 numbered pages. It is your responsibility to ensure that, at the beginning of the exam, all of these pages are included.
2. If you need extra space for any question, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of your continuing work.
3. You may use one standard hand-held calculator that does not perform any: graphing, matrix operations, numerical/symbolic differentiation or integration. All other electronic devices, extra paper, notes, textbooks, pencil/pen carrying cases, food, and drinks in paper cups/boxes or with a label, are forbidden at your workspace either by accident or intent.
4. You may write in pencil, pen, or other ink.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	8	**TOTAL**
30	15	13	14	15	15	21	11	16	150

The following may be helpful:

$$NPV = (\sum PV) - \text{Initial} \qquad S = Pe^{rt} \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$S = P(1+r)^n \quad S = R\left[\frac{(1+r)^{n+1} - 1}{r}\right] - R \qquad A = R + R\left[\frac{1-(1+r)^{-n+1}}{r}\right] \qquad \eta = \frac{p/q}{dp/dq}$$

Part A (Multiple Choice Questions) For each of the following, **clearly print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 3 points and no answer/wrong answers earn 0 points. No justification is required.

1. To three decimals, what annual rate of interest compounding semi-annually is equivalent to 2.6% compounding weekly? (52 weeks = 1 year)

   (A) 2.593% (B) 2.602% (C) 2.610% (D) 2.616% (E) 2.704%

2. If $y = \left(\frac{9}{x^2}\right)^x$ then $y'(3)$ equals

   (A) 3 (B) -2 (C) $\frac{2}{9}$ (D) 2 (E) none of the above

3. If $x \in [1, 3]$, then the smallest value of $f(x) = x + \frac{32}{x^2}$ is

   (A) $\frac{59}{9}$ (B) $\frac{91}{27}$ (C) 33 (D) 6 (E) 10

4. The value of $\int_1^e \frac{2x^3+x}{x^2}\,dx$ is

(A) $1+e^2$ (B) $1+2e^2$ (C) $2e^2$ (D) e^2 (E) $2+e^2$

5. What approximate amount will \$1,200 accumulate to in 20 years if it is invested now at an effective rate of 2.4% per year?

(A) $\$1,906.70$ (B) $\$1,938.36$ (C) $\$1,930.26$ (D) $\$1,928.33$ (E) $\$1,918.62$

6. If $f'(x) = \sqrt{2x+9}$ and $f(-4) = \frac{4}{3}$ then $f(0)$ equals

(A) 1 (B) $\frac{10}{3}$ (C) $\frac{26}{3}$ (D) 10 (E) none of (A) - (D)

7. If $h(x) = xe^{-ax}$ where $a < 0$ is a constant, then we may conclude that h has a

(A) local minimum at $x = \frac{1}{a}$ (B) local maximum at $x = \frac{1}{a}$

(C) local minimum at $x = -\frac{1}{a}$ (D) local maximum at $x = -\frac{1}{a}$

8. If $\int h(x)\,dx = x\,ln(x) + c$ then $h'(e)$ equals

(A) 1 (B) $\frac{e^2}{4}$ (C) $-\frac{1}{e^2}$ (D) $\frac{1}{e}$ (E) e (F) none of (A) - (E)

9. If c is a positive constant, then the area of the region enclosed by the x-axis and the curve $y = -x^2 + 2cx$ is (A) $\frac{4c^3}{3}$ (B) $\frac{16c^3}{3}$ (C) $\frac{4}{3}$ (D) $\frac{2c^3}{3}$ (E) $\frac{8}{3}$

(F) impossible to find because we do not know the exact value of c.

10. Exactly how many of the following statements are always true?

(i) If $f(0) > 0$ and f is continuous at 0, then f is differentiable at 0.

(ii) If the graph of a function $y = f(x)$ is a straight line, then $\int_0^1 f(x)\,dx \geq 0$.

(iii) A continuous function h has a local minimum at a number c if and only if c is a critical value of h.

(iv) If $G_1(x)$ and $G_2(x)$ are antiderivatives of a continuous function $g(x)$, then $G_1(x) = G_2(x)$.

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

Be sure you have printed the letters for your answers in the boxes on the first page

Part B (Full-Solution Questions) Write clear solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32.

1. The parts of this question are independent of each other.

   (a) Find all values of the real constant w such that the function $y = e^{wx}$ is a solution to the equation $y''' + 5y'' - 6y' = 0$. [7 points]

   (b) Find the equation of the tangent line to the curve $y = x^2 - 2x$ that passes through the point $(\frac{3}{2}, -3)$ and has a negative slope. [8 points]

2. The parts of this question are independent of each other.

(a) Evaluate and simplify the expression $\dfrac{x-1}{2x\sqrt{x}} - u'(x)$ where $u(x) = \dfrac{1}{\sqrt{x}} + \sqrt{x}$ and $x \in (0, 5)$. [6 points]

(b) Find $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} f(e + k\Delta x)\Delta x$ where $f(x) = \dfrac{1}{x ln(x)}$ and $\Delta x = \dfrac{e^4 - e}{n}$.

Show all work in your solution. [7 points]

3. The parts of this question are independent of each other.

(a) Assume interest is 3% APR compounding semi-annually. Suppose an investment in a business will result in cash flows consisting of three payments made at the end of 2, 4, and 6 years of amounts 1, 2, and 4, respectively (in units of $\$10,000$). Calculate the largest investment (rounded down to the nearest $\$100$) in the business that is needed to guarantee that the investment is profitable. [7 points]

(b) Assume interest is 4% APR compounding annually. Find the present value of a generalized annuity consisting of $\$2,500$ paid at the end of each year for five years, and $\$4,000$ paid thereafter at the end of each year for three years. Round your final answer up to the nearest dollar. [7 points]

4. (a) Evaluate $\int \frac{x^3}{\sqrt{4+x^2}}\,dx$ [8 points]

(b) Find the exact value (i.e. no decimals) of $\int_1^e 2^{x\,ln(x)}\Big(1+ln(x)\Big)\,dx$ [7 points]

5. In all of this question, let $q \geq 0$ represent quantity and let $\dfrac{dr}{dq} = 12\sqrt{q} + 400e^{-2q} + 3$ be a marginal revenue function. Assume $r = pq$ where p represents the unit price.

(a) Is the marginal revenue increasing or decreasing at $q = 4$? Justify your solution. [4 points]

(b) Find the demand function $p = f(q)$ where $q > 0$. You may assume $r(0) = 0$. [11 points]

6. In all of this question let $\mathcal{R}$ represent the enclosed region in the xy-plane whose boundaries are: the x-axis, the line $y = 2$, the line $y = h(x)$ (where $h(-1) = 0$ and $h(0) = 2$), and the curve $x = y^2 - 2y + 2$.

(a) Give an accurate, labeled sketch of the region $\mathcal{R}$. [5 points]

(b) Find the exact area (i.e. no decimals) of the region $\mathcal{R}$. Show all work. [8 points]

(c) Let $y = c$ be the line that divides $\mathcal{R}$ into two regions of equal area. If we were to use Newton's Method to approximate the number c, write down a formula for obtaining the approximation x_{n+1} from the previous approximation x_n. For this, you will need to state a function $f(x)$ whose root is c. (You do not need to simplify your formula for x_{n+1} and you do not actually need to find the approximation for c.) [8 points]

7. The top and bottom margins of a small poster are $6\,cm$ and the side margins are each $4\,cm$. The actual photographed material on the poster has a fixed rectangular area of $864\,cm^2$ and is surrounded by the margins. Find the dimensions of the poster with the smallest area. Show all work and justification in your solution. [11 points]

8. In all of this question let $f(x) = x\sqrt{5-x}$

(a) State the domain and all intercepts of f. [3 points]

(b) Find $f'(x)$ in factored form and state the intervals where f is increasing or decreasing. [6 points]

(c) Find all local extrema of f. Sufficiently justify your findings. [3 points]

(d) Make a good, labelled sketch of the graph of $y = f(x)$. [4 points]

(This page is intentionally left blank)